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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

Vol. LX. No. 3. January 1900.

INVITATION OF LADIES TO THE ORDINARY MEETINGS OF THE SOCIETY.

The attention of Fellows is called to the fact that Ladies are only admitted to the Ordinary Meetings of the Society by special invitation of the President, sanctioned by the Council. The invitations are issued at the commencement of each Session.

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ROYAL ASTRONOMICAL SOCIETY.

VOL. LX. JANUARY 12, 1900. No. 3

Professor G. H. DARWIN, M.A., LL.D., PRESIDENT, in the Chair.

Professor R. N. Apte, M.A., LL.B., Rajaram College, Kolhapur, India ;
Maurice Harvey Clarke, Lieut. R.N.R., F.R.G.S, Coleswood, Harpenden, Herts ;
Rev. P. W. Fairclough, Kaiapoi, New Zealand ;
Umes Chandra Ghosh, M.A., Muir Central College, Allahabad, India ;
Richard Kilvington Hattersley, 4 Church Terrace, Blackheath, S.E. ; and
Howard Payn, F.R.G.S., 21 Hyde Park Place, W.,

were balloted for and duly elected Fellows of the Society.

Sixty-six presents were announced as having been received since the last meeting, including amongst others :—

L. Ambronn, Handbuch der Astronomischen Instrumentenkunde, presented by the author ; Nice, Observatoire, Annales, Tome i., presented by M. Bischoffsheim ; Paris, Observatoire, Carte Photographique du Ciel, Zone 24 (19 charts), presented by the Observatory ; Isaac Roberts, Photographs of stars, star clusters and nebulæ, vol. ii., presented by the author ; S. Newcomb, Tables of *Uranus* and *Neptune*, presented by the Office of the American Ephemeris ; Photograph of the nebula H V. 14 *Cygni*, presented by Mr. Wilson ; Photograph of the nebula in *Orion*, presented by the Astronomer Royal.

The Determination of Selenographic Positions and the Measurement of Lunar Photographs. By S. A. Saunder, M.A..

The object of this paper is to call attention to the great uncertainty that attaches to our knowledge of the positions of lunar formations, and to suggest means by which these positions may be determined more accurately than has yet been accomplished.

§ 1. *Errors in Existing Measures.*

Schmidt, in the introduction to his *Charte der Gebirge des Mondes*, gives a list of 157 formations whose positions have been determined as "points of the first order," almost entirely by Lohrmann (76), Mädler (89), and Neison (33). If to these we add the positions of Biot, found by Mädler; Horrocks, Halley, and Hipparchus L, by Neison; with Triesnecker B, by Pritchard, we have, so far as I know, all the points whose positions have been found by methods making any pretension to accuracy; and of these very little reliance can be placed on those which depend on not more than two or three measures. The uncertainty attaching to a single measure was estimated by Mädler* to be about 8″ or 9″ (geocentric), which would correspond to half a degree of selenographic longitude or latitude near the centre of the disc, and to a still greater amount as the limb is approached. The probable error derived from the measures themselves, *after rejecting the unfavourable points*, is stated by Neison† to be 7″·2. Rather less than half the whole number given by Schmidt depend upon 10 or more measures, whether by the same or by different observers.

Eleven points were measured five times at least by both Lohrmann and Mädler. The average difference in selenographic latitude is 17′, and in longitude 10′, corresponding respectively to 4″·4 and 2″·3 geocentric. It may also be noted that in nine cases Lohrmann's position is further from the equator than Mädler's, and the mean distance of his positions from the equator is greater than Mädler's by 12′ of selenographic latitude, or 3″·4 as seen from the Earth. The differences in longitude so far as these points are concerned do not appear to be systematic.

§ 2. *Possible Cause of Systematic Error.*

The method adopted by Lohrmann, Mädler, and Neison was that suggested by Encke, in which the position of a point is determined by measuring its distance from either the N. or S.

* *Der Mond*, § 39.

† *Quarterly Journal of Science*, April 1875.

limb, and also from the E. or W., according to the state of illumination. Lohrmann found the diameter of the Moon by measuring the line joining the cusps ; Mädler by computation from Encke's ephemeris.

A personal equation is known to exist in observations of the Moon's limb, and no indication of the error due to this cause is given by the probable error of a series of measures obtained from those measures themselves, whilst in comparing one observer with another we find only the difference in the extent to which the two are affected by this or any other systematic error, so that the real errors may be much larger than would be inferred from this probable error, combined with the result of such a comparison.

I think we may see one direction in which systematic error is likely to occur.

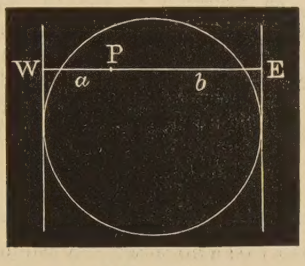


Fig. 1.

Let P be a point whose position is to be found.

Let PW$=a$, PE$=b$, be its distances from the west and east limbs respectively.

Let $2r$ be the Moon's diameter.

All these quantities will in consequence of irradiation be measured too great. Suppose there is a definite increment or personal equation, due to this cause, and call it x. The measured diameter is increased by $2x$, the radius by x, so that, if we measure from the west limb, instead of determining $\frac{a}{r}$ we really measure $\frac{a+x}{r+x}$, and as $a<r$, it follows that $\frac{a+x}{r+x}>\frac{a}{r}$, and the resulting position is too near the central meridian.

Mädler and Neison seldom measured a point from both limbs, Lohrmann usually did so ; but if P were measured from the east limb we should measure $\frac{b+x}{r+x}$ instead of $\frac{b}{r}$, and as $b>r$, it follows that $\frac{b+x}{r+x}<\frac{b}{r}$, and again the point is brought too near the central meridian.

Mädler's computed radius was also affected by irradiation, but the assumption that the increments of the numerator and denominator have a tendency to equality becomes less probable in his case.

Measures from the north or south limb would be similarly affected, and we might expect measures made in this way to give latitudes and longitudes numerically too small. I shall offer some evidence that this has been the case, and suggest that this is a real, though I believe not the only, cause of systematic error.

§ 3. *Proposed Substitution of Measures from a known Point.*

The great difficulty of measuring from the limb has always been recognised ; it has been suggested that transits of a well-defined formation, as Mösting A, should be observed in preference to transits of the limb, and tables are given in the *Berliner Jahrbuch* to facilitate their reduction. Neison proposed that measures for selenographic positions should similarly be taken from a well-determined point ; but so far as I am aware neither he nor anyone else has ever published an account of measures made in this way.

§ 4. *Correction of the Measures for Libration.*

I expect that the real reason why so few selenographers have given attention to this branch of the work is to be found in the very forbidding character of the reductions necessitated by the Moon's librations. This certainly would have prevented my ever taking it up had it not been that I mentioned the difficulty to Professor Turner, who suggested the following elegant method of shortening the computation by the use of rectangular coordinates :—

Let ξ, η, ζ be the "standard" coordinates of any point referred to rectangular axes through the Moon's centre, the axis of η being the Moon's polar axis, that of ζ the radius through the intersection of the equator with the first meridian.

Let x, y, z be the "instantaneous" coordinates of the same point, the axis of y being the projection of the Moon's central meridian on the plane normal to the line of sight ; that of z the line of sight through the centre of the disc ; so that the "instantaneous" coincide with the "standard" coordinates under mean libration.

The ordinary formulæ for the transformation of coordinates give

$$\left.\begin{aligned} x &= a_1\xi + b_1\eta + c_1\zeta \\ y &= a_2\xi + b_2\eta + c_2\zeta \\ z &= a_3\xi + b_3\eta + c_3\zeta \end{aligned}\right\} \quad (1)$$

and for the reverse transformation

$$\left.\begin{aligned}\xi &= a_1x + a_2y + a_3z\\ \eta &= b_1x + b_2y + b_3z\\ \zeta &= c_1x + c_2y + c_3z\end{aligned}\right\} \quad (2)$$

If l', b' be the Moon's librations in longitude and latitude respectively it is easily shown that

$$\left.\begin{aligned} a_1 &= \cos l', \quad a_2 = -\sin l' \sin b', \quad a_3 = \sin l' \cos b'\\ b_1 &= 0, \qquad b_2 = \cos b', \qquad\quad b_3 = \sin b'\\ c_1 &= -\sin l', \; c_2 = -\cos l' \sin b', \quad c_3 = \cos l' \cos b'\end{aligned}\right\} \quad (3)$$

Now take the Moon's radius as the unit of length.

Let d=the distance of the Moon's centre from the observer,

s'=the Moon's angular semidiameter;

then
$$d = \frac{1}{\sin s'},$$

and the equations to the line of sight which passes through

(a) the position of the observer, 0, 0, d,

(b) the point observed, x, y, z,

are
$$\frac{X}{x} = \frac{y}{y} = \frac{Z-d}{z-d}.$$

This meets the plane of projection, Z=0, at the point whose coordinates are

$$\frac{xd}{d-z}, \quad \frac{yd}{d-z}, \quad 0$$

or
$$\frac{x}{1-z\sin s'}, \quad \frac{y}{1-\sin s'}, \quad 0.$$

Let Q be the measured distance between two points (x, y, z), $(x', y' z')$, expressed as a fraction of the Moon's radius.

P, the position angle of the direction of Q from (x, y, z) to (x', y', z').

C, the position angle of the axis of y.

$C-\frac{\pi}{2}$ is the position angle of the axis of x;

and we have

$$\frac{x'}{1-z'\sin s'} - \frac{x}{1-z\sin s'} = Q\cos\left(P-C+\frac{\pi}{2}\right) = -Q\sin(P-C).$$

$$\frac{y'}{1-z'\sin s'} - \frac{y}{1-z\sin s'} = Q\sin\left(P-C+\frac{\pi}{2}\right) = Q\cos(P-C).$$

These equations may be written

$$\left.\begin{aligned} x_1'-x &= -Q\sin(P-C)(1-z\sin s') \\ y_1'-y &= Q\cos(P-C)(1-z\sin s') \end{aligned}\right\} \quad (4)$$

where $x'=x_1'(1-z'\sin s')(1-z\sin s')^{-1}$

$$=x_1'\{1-(z'-z)\sin s'-z(z'-z)\sin^2 s'+\ \ldots\}$$

$$\left.\begin{aligned} &=x_1'\{1-(z'-z)\sin s'(1+z\sin s')\}\ \text{approximately} \\ \text{Similarly}\quad &y'=y_1'\{1-(z'-z)\sin s'(1+z\sin s')\} \qquad ,, \end{aligned}\right\} \quad (5)$$

Since $\sin s'<\cdot 005$, this approximation is always close enough; and, unless $z'-z$ is large, the term in $\sin^2 s'$ may also be rejected.

If (x, y, z) be the point whose position is known, equations (4) give us the values of x_1', y_1'.

From these may be found $z_1'=\sqrt{1-x_1'^2-y_1'^2}$.

And if we use this, instead of z', in equations (5), we can find x', y'.

The error introduced in x' by using this approximate value of z is sensibly $x_1'(z_1'-z')\sin s'$.

From equations (5) we see that

$$x_1'-x'=x_1'(z'-z)\sin s' \text{ approximately};$$

$$y_1'-y'=y_1'(z_1-z)\sin s' \qquad ,, \qquad ;$$

whence $$z_1'-z'=\frac{x_1'^2+y_1'^2}{z_1'}(z'-z)\sin s';$$

and the error in x' becomes $x_1'\dfrac{x_1'^2+y_1'^2}{z_1'}(z'-z)\sin^2 s'$ (6)

Unless we are working close to the limb, this will be insensible. If necessary its value may be computed, again using z_1' instead of z'; but I have never had to do this.

The computation may therefore be arranged as follows:—

(*a*) Find x, y, z, the "instantaneous" coordinates of the known point by (1).

(*b*) Find x_1', y_1' by (4).

(*c*) Find $z_1'=\sqrt{1-x_1'^2-y_1'^2}$.

(*d*) Find x', y' by (5), using z_1' instead of z'.

(*e*) Find $z'=\sqrt{1-x'^2-y'^2}$.

(*f*) Find ξ', η', ζ', the "standard" coordinates of the unknown point, by (2).

If the selenographic longitude and latitude are required they can be found from $\sin(\text{lat.})=\eta'$, $\cos(\text{lat.})\sin(\text{long.})=\xi'$.

Note that when z' has been computed by (*e*) we can at once form the value of $z'-z$, and determine whether the residual (6) is sensible.

Quite apart from the shortening of the work, Professor Turner's method has the very important advantage that, beyond finding the sine and cosine of P—C, no reference to any trigonometrical tables is required. No attempt has been made to adapt the work to logarithmic computation. As it stands, the operations are just those in which the "Brunsviga" excels, and by the use of this machine the computation becomes purely mechanical.

§ 5. *Computation of the Librations.*

It remains to explain the method adopted in computing the librations. As was pointed out by Neison, these need not be so exactly determined as was necessary in Encke's method, but an approximate correction for parallax is required.

The following tables are used :—

(1) The values of N, log cot n and log sin n, as proposed by Gauss, where

$$\tan \mathrm{N} = \cot \phi \cos \tau$$

$$\cot n = \sin \mathrm{N} \tan \tau$$

τ between the hour angle and ϕ the latitude.

These are tabulated for every minute with argument τ, and are used in correcting for refraction, and also in determining the Moon's apparent altitude from

$$\sin (\text{alt.}) = \sin n \sin (\mathrm{N} + \delta').$$

(2) A table constructed on the same principle for converting right ascension and declination into latitude and longitude. A very complete table of this form, with corrections for variations in obliquity of the ecliptic is given in Hansen's *Tables de la Lune*, but the simpler form, given in the *American Ephemeris* from 1855 to 1864 for mean obliquity, is all that is necessary.

(3) Tables giving (1) $\frac{\alpha - \alpha'}{\pi}$ the parallax in right ascension as a fraction of the horizontal parallax ;

(2) $\frac{\delta - \delta'}{\pi}$ the parallax in declination as a fraction of the horizontal parallax

computed from the equations

$$\frac{\alpha - \alpha'}{\pi} = \rho \cos \phi' \sin t \sec \delta$$

$$\tan \gamma = \tan \phi' \sec t$$

$$\frac{\delta - \delta'}{\pi} = \rho \sin \phi' \sin (\gamma - \delta) \operatorname{cosec} \gamma,$$

(see Chauvenet, vol. i. art. 101) and tabulated with the double arguments, α and δ.

These formulæ are not rigorous, but the parallax found from them is quite near enough.

(4) A table giving the augmentation of the Moon's semi-diameter with the apparent altitude as argument.

It is also useful to have tables for converting minutes and seconds of arc, as well as time, into the decimals of a degree.

The geocentric librations, l, b, are taken from the very useful ephemeris prepared by Mr. Crommelin, for which the thanks of all selenographers are due to him.

The parallax in α and δ is taken from tables (3), the necessary data being given in the *Nautical Almanac*. The apparent α' and δ' are converted in longitude and latitude λ', β' by tables (2).

The geocentric longitude and latitude λ, β may be similarly computed and compared with the values given in the *Nautical Almanac* for verification.

The corrected librations l', b' are then found from

$$l'=l-(\lambda-\lambda') \qquad b'=b+(\beta-\beta')$$

which, though not rigorous, are sufficiently close.

The Moon's apparent altitude is computed by table (1), and another verification is found as follows: The total parallax is added to the apparent altitude, giving the geocentric altitude. From this the co-latitude is subtracted, and, treating the remainder as a declination, the corresponding value of $\frac{\delta-\delta'}{\pi}$, when $\alpha=0$, is taken from table (3). This, multiplied by π, the horizontal parallax, should again give, very approximately, the total parallax.

The value of s, given in the *Nautical Almanac*, is corrected for altitude by table (4); the result corresponds to the "bright radius," and must be reduced by $1''{\cdot}5$ in order that Q may be expressed as a fraction of the "dark" or occultation radius, as determined by Dr. L. Struve from the lunar eclipses of 1884 and 1888. I do not think there can be any question that this radius should be employed in preference to the "bright" radius affected by irradiation.

Finally the value of C is computed by each of the formulæ given in the preface to the *Nautical Almanac*, the agreement serving as a further verification.

§ 6. *Method and Results of Observation.*

The reductions are still long, and it is therefore desirable to make the observations with care, the mean of several measures made consecutively taking no longer to reduce than a single measure. My usual method in the most recent work has been to take three settings for position angle, turn the position circle through 180° and take three more; then to take three readings

for distance with the webs open, *i.e.* each on the same side of the field as its own screw head ; five with the webs crossed, or each nearer to the opposite screw head ; and two more with the webs open ; finally to take six more settings for position angle as before. In this way the mean time for the position angle becomes nearly the same as that for the distance. If this condition is not fulfilled a correction to the position angle may be required, as when the Moon's right ascension is near 6 hours or 18 hours the value of C changes by about 15′ an hour.

The whole set takes from half to three quarters of an hour. This may seem long, but measuring on the Moon has difficulties of its own in addition to those encountered in estimating the centres of the formations. Owing to the varying motion in right ascension it is seldom that the clock is exactly adjusted, nor is it worth while to attempt more than an approximate adjustment, for there is always the motion in declination outstanding. This makes it far from easy to fix the webs on two distant points, and to verify the setting. The greater the distance the greater the difficulty. The change in libration during the interval is not sufficient to sensibly affect the measures, and a few points accurately measured are of more value than a greater number less carefully obtained.

Most of the work I have done by this method has been directed to fixing the positions of the smaller formations in a restricted region, but it may be also applied to determining the more important points.

The only point on the Moon whose position is known with real accuracy is Mösting A, as determined by Professor Franz from Schlüter's observations with the Königsberg heliometer. These were made under Bessel's direction for the determination of the Moon's physical libration. Mösting A was the point observed, and its resulting position, as given in an appendix to vol. xxxviii. of the *Königsberg Observations*, is

$$\lambda = -5^\circ\ 10'\ 19''{\cdot}0 \pm 7''{\cdot}9 \quad \beta = -3^\circ\ 11'\ 24''{\cdot}0 \pm 5''{\cdot}5.$$

The following measures, made and reduced as described above, give a fair idea of the accuracy with which positions can be obtained by this method. They are selected as being useful for comparison with results obtained from photographic measures in the second part of this paper, but at present I call attention only to the mutual agreement of different measures of the same point.

Ptolemaus, A. (*Centre.*)

		ξ	η
1898	Sept. 30	$-{\cdot}0141$	$-{\cdot}1484$
	Dec. 22	$-{\cdot}0139$	$-{\cdot}1476$
	23	$-{\cdot}0138$	$-{\cdot}1476$
1899	Jan. 23	$-{\cdot}0136$	$-{\cdot}1475$
	Feb. 19	$-{\cdot}0141$	$-{\cdot}1475$
	Mean	$-{\cdot}0139$	$-{\cdot}1477$

Herschel. (*Summit of cone.*)

1898 Dec.	22	$\xi = -\cdot0370$	$\eta = -\cdot0988$
	23	$-\cdot0370$	$-\cdot0993$
1899 Feb	19	$-\cdot0371$	$-\cdot0989$
	Mean	$-\cdot0370$	$-\cdot0990$

Dollond. (*Centre.*)

1899 Nov.	13	$\xi = +\cdot2460$	$\eta = -\cdot1817$
	16	$+\cdot2458$	$-\cdot1813$
	18	$+\cdot2455$	$-\cdot1818$
Dec.	11	$+\cdot2459$	$-\cdot1820$
	Mean	$+\cdot2458$	$-\cdot1817$

Manilius. (*Summit of cone.*)

1899 Nov.	13	$\xi = +\cdot1544$	$\eta = +\cdot2497$
Dec.	11	$+\cdot1531$	$+\cdot2493$
	13	$+\cdot1537$	$+\cdot2497$
	Mean	$+\cdot1537$	$+\cdot2496$

Hesiodus B. (*Centre.*)

1899 Dec.	14*	$\xi = -\cdot2670$	$\eta = -\cdot4553$
	15	$-\cdot2673$	$-\cdot4553$
	Mean	$-\cdot2672$	$-\cdot4553$

Euclides. (*Centre.*)

1899 Dec.	13	$\xi = -\cdot4886$	$\eta = -\cdot1279$
	15	$-\cdot4887$	$-\cdot1283$
	Mean	$-\cdot4887$	-1281

These positions all carry with them any error in that of Mösting A, but this, I believe, is a very small quantity.

When it is remembered that the last place here given has a geocentric value of something less than $0''\cdot1$, and that the probable error of either coordinate from one night's work deduced from these results is $\cdot00017$, or about $0''\cdot16$ (geocentric), whilst no divergence from the mean exceeds $0''\cdot7$, it will be seen that the agreement is very different from that attained by Encke's method (see §1).

* An incomplete set, stopped by clouds.

§ 7. *Measurement of Photographs.*

The results I was obtaining by this method offered such great promise that last Easter I again consulted Professor Turner as to the possibility of applying it to the measurement of lunar photographs. He entered most cordially into the idea, and in addition to giving many valuable suggestions as to methods, he promised to lend me a micrometer, and wrote to M. Loewy asking for photographs on my behalf.

M. Loewy, with great generosity, at once sent four of his incomparable negatives, with terminators at about $+14°$ M., $-11°$ M., $+18·5$ E., $-7·5$ E., so that all the principal formations are well shown, and Mr. Bellamy, of the Oxford University Observatory, very kindly made positive copies on glass of the second and third of these with *réseaux*. This proved a more difficult operation than I had anticipated. When the copies were dense enough to bring out the detail the *réseau* was lost in the shadows and on the darker parts of the surface. Eventually successful copies were obtained, but only after much careful and patient work, for which, as well as for the kindness and heartiness with which it was undertaken, I owe him my warmest acknowledgments.

I was not able to commence work upon the plates until August, and in the meantime Mr. Bellamy had measured thirty-six points of the first order on a copy of the third plate. I subsequently found approximate values for the radius and coordinates of the centre, and then proceeded to determine constants for the plate on a method suggested by Professor Turner, analogous to that devised by him for the reduction of the astrographic chart plates.

§ 8. *Correction for Finite Distance.*

The first thing to be done is from the measured, conically projected, values of the *réseau* coordinates $x+\delta x, y+\delta y$, to deduce the orthogonally projected coordinates x, y of the same point. In other words we have to correct for the finite distance of the Moon.

The axes of x, y being in the same plane as in § 4, we have as there shown

$$x=(x+\delta x)\,(1-z\sin s') \qquad y=(y+\delta y)\,(1-z\sin s').$$

or $$\delta x=(x+\delta x)\,z\sin s' \qquad dy=(y+\delta y)\,z\sin s'$$

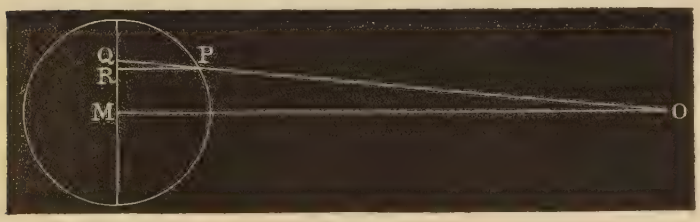


Fig. 2.

If O be the position of the observer,
P the point observed,
M the centre of the Moon,
Q, R the conical and orthogonal projections of P respectively,
Let the radius of the Moon be the unit of length :

$$MR=r \quad MQ=r+\delta r \quad PR=z=\sqrt{1-r^2}$$

and let the corresponding value of z for the point at which the ordinate through Q meets the sphere be $z+\delta z$, so that $z+\delta z=\sqrt{1-(r+\delta r)^2}$.

Then, since δr, δz are small,

$$r\delta r=-z\delta z.$$

From the values just found for δx, δy we have

$$\begin{aligned}\delta r&=(r+\delta r)z\sin s'\\ \therefore \delta z&=-\frac{r}{z}\delta r\\ &=-r(r+\delta r)\sin s'\\ &=-(r+\delta r)^2\sin s' \text{ approximately.}\end{aligned}$$

Hence the required corrections may be written

$$\begin{aligned}\delta x&=(x+\delta x)\{(z+\delta z)\sin s'-\delta z\sin s'\}\\ &=(x+\delta x)\{(z+\delta z)\sin s'+(r+\delta r)^2\sin^2 s'\}.\end{aligned}$$

Similarly

$$\delta y=(y+\delta y)\{(z+\delta z)\sin s'+(r+\delta r)^2\sin^2 s'\}.$$

These may be computed from the measured values $x+\delta x$, $y+\delta y$, the terms in $\sin^2 s'$ being always less than ·000025.

§ 9. *Failure of the First Attempt to Determine Constants for the Plate.*

The further corrections to be applied to x, y are those for errors of centre and scale, for refraction and for optical distortion. These with the transformation necessary to deduce the values of the "standard" coordinates ξ, η, ζ are all linear, and therefore we have

$$\begin{aligned}x&=A\xi+B\eta+C\zeta+D\\ y&=E\xi+F\eta+G\zeta+H,\end{aligned}$$

where A, B . . . H are constants for the plate ; and if these are known, the equations are sufficient to determine ξ, η, ζ, which are connected by the relation $\xi^2+\eta^2+\zeta^2=1$.

In order to determine the constants, the accepted coordinates of the points measured by Mr. Bellamy were substituted with the corresponding values of x, y, giving a group of thirty-six equations for A, B, C, D, and thirty-six for E, F, G, H, the two groups differing only in the absolute terms x, y.

When the solution was effected by least squares, the probable errors of C, D were ·0043, ·0041 respectively, and the mean of the eight probable errors was ·0022. The probable error in the longitude of the centre therefore corresponded to more than 5″ (geocentric), whilst the positions obtained for Mösting A and one or two of the best known points were between 2″ and 3″ (geocentric) too far east. The residuals from the conditional equations in x (*i.e.* the value of $A\xi+B\eta+C\zeta-x$) ranged from $-·0128=-12''·0$ geocentric to $+·0074=+6''·9$ geocentric, the numerical mean being $·0029=2''·7$, the corresponding mean for the y group being $·0020=1''·9$, showing, as is apparent all through the work, that the latitudes have been better determined than the longitudes. But these residuals were too large to be accounted for by errors in the photographic measures, and the constants were so badly determined that the only conclusion was that the accepted places were not good enough for what was required of them.

Though disappointing in the immediate result, this was in other ways encouraging. It showed that a revision of the places was necessary, and that the photographic measures were adequate to effect it, if a method could be devised to make them yield the information they contained.

§ 10. *General Principle of the Solution.*

Eventually the following was adopted.

The time and place at which each photograph was taken being known, all the information required as to scale, position of centre, and plane of projection could be obtained partly by computation, partly by measures on the photograph, but the orientation could not be so obtained.

It was noted that the second and third of the photographs sent by M. Loewy, taken a little after first and a little before third quarter respectively, were under very different librations. When these librations were computed, the absolute position of the centre of the disc in each photograph became known; and if these two positions could be combined, we should have a direction from which the orientations might be obtained. But it was doubtful whether the distance between the centres would prove sufficient to fix this direction with the required accuracy.

A pretty close physical interpretation of the process adopted may be given as follows:—

The two plane pictures were by no means superposable, and a difference in configuration on the common zone was easily

seen over quite a small area. If, then, these two pictures could be put into a stereoscope—as De la Rue did—they would give a solid picture of the Moon on which the relative positions of the formations could be observed ; but there is only one way in which they could be put into the stereoscope, so that combination by the eyes might be possible. Again, the knowledge of the librations gave us the position of the plane of projection referred to the equator and first meridian, and so it would be possible to construct for each picture a corresponding projection of a set of meridians and parallels of latitude which might be fitted on to the picture if only we knew how to orientate it. If these two projections were put into the stereoscope, without the Moon pictures, there would again be only one way of doing it which would enable the two to be combined by the eyes. If these projections were made on glass, so that the photographs of the Moon could be seen through them, and all four pictures were put into the stereoscope together, we should, when all four were adjusted, see the meridians and parallels on the Moon itself, and could read off the latitude and longitude of any desired point. And, further, as we should have thus found how to place the meridians and parallels on each picture, we could take them out of the stereoscope and obtain a separate reading of the position of any point on each photograph, or determine the position of a point shown on one plate only.

§11. *First Approximation to the Constants of the Plates.*

The analytical process was as follows. The optical librations were carefully computed with Franz's value of the inclination of the Moon's equator, a rigorous formula being used for the parallax. To these were added the physical librations taken from the *Berliner Jahrbuch*, the signs there given being reversed for this purpose, as noted by Mr. Crommelin in his last Ephemeris. The results were :—

Reference Number.	Date.	Librations.	Approx. longitude of Terminator.
II.	1899, Feb. $18^d\ 6^h\ 30^m\ 14^s$, Paris sid. t.	$l' = +5°\ 1'{\cdot}7$, $b = -1°\ 57'{\cdot}2$	$-11°$ M.
III.	1895, Sept. $9^d\ 1^h\ 38^m\ 2^s$, ,, ,,	$-6°34'{\cdot}4$, $-5°\ 53'{\cdot}6$	$+18°{\cdot}5$ E

from which it follows that the selenocentric distance between the centres of the two discs is $12°\ 13'$.

The zenith distances and the parallactic angles at the centres of the discs were computed at the same time.

Professor Turner has kindly allowed me to see the unpublished MS. of Professor Pritchard's measures of photographs made in his investigation of the Moon's physical libration. From these I find that he obtained the following positions : —

Ptolemaus A, $\xi = -{\cdot}0144$, $\eta = -{\cdot}1483$ from 60 photographs.
Triesnecker B, $\xi = +{\cdot}0066$, $\eta = +{\cdot}0202$,, 52 ,,

The probable error of Professor Pritchard's determination of Ptolemaus A was a little greater than that of mine as given in § 6, but the two positions differ only by about half a second (geocentric) in each coordinate, and the means $\xi = -\cdot 0142$, $\eta = -\cdot 1480$, may be taken as very nearly accurate.

These, with Professor Franz's position of Mösting A, $\xi = -\cdot 0900$, $\eta = -\cdot 0556$ gave three points whose positions seemed trustworthy, and which were employed as stated below.

If x, y be the *réseau* coordinates of any point corrected for finite distance and refraction, referred to the centre, and expressed as fractions of the Moon's radius; and z is the third coordinate of the same point formed from the equation $x^2+y^2+z^2=1$; ξ, η, ζ the "standard" coordinates of the same point; θ the angle through which the *réseau* axis of y must be turned to bring it into coincidence with the projection of the Moon's central meridian, the positive direction of rotation being the same as for position angles, the ordinary formulæ for transformation of coordinates give

$$\xi = (\cos l' \cos\theta - \sin l' \sin b' \sin\theta)x - (\cos l' \sin\theta + \sin l' \sin b' \cos\theta)y + \cos b' \sin l' z$$

$$\eta = \cos b' \sin\theta\, x + \cos b' \cos\theta\, y + \sin b' z$$

which may be written

$$\left.\begin{aligned} \xi &= Ax + By + Cz \\ \eta &= Dx + Ey + Fz \end{aligned}\right\} \quad (7)$$

And the problem is to determine the value of θ.

In Plate II. 39 points on the limb were measured, and in Plate III. 36 points.

A selection from these gave approximate values of the coordinates of the centre and of the Moon's radius, in *réseau* intervals.

The *réseau* coordinates of the three known points were then measured, corrected for everything except refraction, and substituted in (7).

From these equations were then deduced an approximate value of θ for each plate.

In expressing the coordinates as fractions of the radius, I assumed that the radius measured on the photograph was the "bright" radius, given in the *Nautical Almanac*, which accords very closely with Professor Pritchard's determination of the Moon's photographic diameter (*Memoirs R.A.S.*, vol. xlvii.)

This was reduced by $1''\cdot 5$ as in § 5.

§ 12. *Coordinates of the Zenith, and Correction for Refraction.*

The coordinates of the zenith were then found from the equations

$$X = \tan\zeta' \sin(C + \theta - q')$$
$$Y = \tan\zeta' \cos(C + \theta - q')$$

where ζ' is the computed apparent zenith distance,
C the position angle of the Moon's central meridian, computed as in § 5,
q' the parallactic angle at the centre of the disc.

The measured coordinates could now be corrected for refraction, these corrections being

$$57''\{(1+X^2)x+XYy\} \text{ to } x \text{ and } 57''\{(1+Y^2)y+XYx\} \text{ to } y.$$
[*Monthly Notices*, vol. liv. p. 19.]

the numerical values of which were

for Plate II., ·0003x to x and ·0003y to y,
for Plate III., ·0003x to x and ·0004y to y,

the second term being insensible in each case.

§ 13. *Determination of Centre and Radius.*

From the whole group of corrected limb measures an attempt was then made to find the final coordinates of the centre and the radius of the disc. Unfortunately the Oxford University Observatory *réseau* was not large enough to cover the whole limb, and from this it resulted that the abscissa and radius were poorly determined. The limb measures were therefore combined with those of the three known points, and the final values deduced from the whole.

The values of the two radii showed that on the scale adopted a side of a *réseau* square on Plate II. corresponded to 57″·57; on Plate III. to 57″·62. Assuming the side to be exactly 5mm. the corresponding focal lengths of the object glass would be 17·915m. and 17·899m. Unfortunately this focal length has never been accurately determined.

It is interesting to note that the figures give a smaller mean radius in Plate II., where we measure from the W. limb, than in Plate III., where we measure from the E. limb, the difference being 0″·8. Professor Pritchard found from his measures of the photographic diameter that the W. limb did give a semidiameter smaller by 0″·2 than the E. limb, so that part of the difference may be due to this cause. As one photograph was taken in February, the other in September, it is probable that there would be some difference in the focal length.

§ 14. *Determination of the Orientation.*

Values had now been obtained for all the constants of the plate except θ, the value of which, deduced from the three known points, was liable to sensible error in consequence of their nearness to the centre.

To determine this, sixty-two points were measured on both plates on the zone common to the two, and all the corrections applied.

Twenty points were rejected on account of difficulty in reading the *réseau*, or unsuitable illumination on one or other of the plates. This left the forty-two points shown in Table I., the measures of which were considered satisfactory.

If a small error has been made in θ, it will be found by differentiating equations (7) that the true values of ξ, η are given by

$$\xi = Ax + By + Cz + (Bx - Ay)\delta\theta$$
$$\eta = Dx + Ey + Fz + (Dx - Ey)\delta\theta$$

If therefore we use the suffixes 2, 3, to denote the plate to which the corresponding quantity belongs, I had to find values of $\delta\theta_2$, $\delta\theta_3$, which would make $\xi_2 = \xi_3$, $\eta_2 = \eta_3$.

This gave forty-two equations of the type

$$(Ax + By + Cz)_2 - (Ax + By + Cz)_3 + (Bx - Ay)_2\,\delta\theta_2 - (Bx - Ay)_3\,\delta\theta_3 = 0$$

and forty-two of the type

$$(Dx + Ey + Fz)_2 - (Dx + Ey + Fz)_3 + (Dx - Ey)_2\,\delta\theta_2 - (Dx - Ey)_3\,\delta\theta_3 = 0.$$

The normal equations derived from the ξ group were

$$\left.\begin{aligned} 9205\delta\theta_2 - 8871\delta\theta_3 + 41{\cdot}70 &= 0 \\ -8871\delta\theta_2 + 8695\delta\theta_3 - 40{\cdot}35 &= 0 \end{aligned}\right\} \quad (8)$$

Those from the η group

$$\left.\begin{aligned} 273\delta\theta_2 + 172\delta\theta_3 - {\cdot}20 &= 0 \\ 172\delta\theta_2 + 714\delta\theta_3 - 2{\cdot}46 &= 0 \end{aligned}\right\} \quad (9)$$

which combine to

$$\left.\begin{aligned} 9478\delta\theta_2 - 8699\delta\theta_3 + 41{\cdot}50 &= 0 \\ -8699\delta\theta_2 + 9409\delta\theta_3 - 42{\cdot}81 &= 0 \end{aligned}\right\} \quad (10)$$

the solution being

$$\left.\begin{aligned} \delta\theta_2 &= -{\cdot}00134 \pm {\cdot}00027 \\ \delta\theta_3 &= +{\cdot}00331 \pm {\cdot}00027 \end{aligned}\right\} \text{ in circular measure.}$$

The resulting positions are given in Table I. for each plate, with columns showing the differences, and the results obtained by previous observers for comparison.

TABLE I.

List of Forty-two Points measured on Plates II. and III. from which the Values of $\delta\theta_2$, $\delta\theta_3$ were deduced.

	ξ_2	ξ_3	$\xi_2-\xi_3$	η_2	η_3	$\eta_2-\eta_3$	Other Observers. ξ	η	Authority.
Lilius	+·0621	+·0610	+·0011	−·8131	−·8141	+·0010			
„ sm. crater near* ...	+·0619	+·0621	− 2	−·7961	−·7966	+ 5			
Licetus *d*	+·0566	+·0554	+ 12	−·7724	−·7729	+ 5			
„ H	+·0383	+·0380	+ 3	−·7182	−·7184	+ 2			
Huggins †	−·0286	−·0301	+ 15	−·6495	−·6498	+ 3			
Stöfler K	+·0567	+·0567	0	−·6353	−·6355	+ 2			
Orontius *d*	−·0829	−·0831	+ 2	−·6346	−·6345	− 1			
Miller	+·0096	+·0098	− 2	−·6339	−·6339	0			
Walter *a* ‡	+·0095	+·0088	+ 7	−·5936	−·5938	+ 2			
„ sm. crater § ...	−·0023	−·0027	+ 4	−·5547	−·5546	− 1			
Lexell	−·0580	−·0588	+ 8	−·5815	−·5818	+ 3			
Aliacensis	+·0758	+·0760	− 2	−·5062	−·5064	+ 2			
Werner central mount. ...	+·0539	+·0544	− 5	−·4689	−·4689	0	+·0502 +·0458	−·4551 −·4658	Lohrmann Mädler
„ N.E. mount. ‖ ...	+·0456	+·0454	+ 2	−·4655	−·4663	+ 8			
Purbach A	−·0295	−·0296	+ 1	−·4406	−·4408	+ 2			

* The most westerly of three confluent craters of which the easternmost is *b*.
† The interior crater in contact with the N.E. wall.
‡ The small isolated crater S. of Walter.
§ The isolated crater on the S.E. part of the floor.
‖ The mountain on the N.E. part of the floor.

	ξ_2	ξ_3	$\xi_2-\xi_3$		η_2	η_3	$\eta_2-\eta_3$		Other Observers. ξ	Other Observers. η	Other Observers. Authority.
Faye	+·0617	+·0620	−	3	−·3651	−·3648	−	3			
Donati	+·0838	+·0837	+	1	−·3523	−·3526	+	3			
Arzachael *c*	+·0199	+·0204	−	5	−·3176	−·3179	+	3			
Airy	+·0955	+·0948	+	7	−·3103	−·3101	−	2			
„ sm. crater *	+·0744	+·0747	−	3	−·3026	−·3025	−	1			
Argelander...	+·0975	+·0974	+	1	−·2843	−·2848	+	5			
Alphonsus	−·0465	−·0457	−	8	−·2311	−·2316	+	5	−·0551	−·2248	Mädler
Albategnius A	+·0444	+·0449	−	5	−·2073	−·2065	−	8			
Ptolemaus A	−·0139	−·0141	+	2	−·1479	−·1479		0	−·0144	−·1483	Pritchard
									−·0168	−·1492	Neison
Hipparchus G	+·1290	+·1292	−	2	−·0871	−·0870	−	1			
„ *n*	+·0874	+·0870	+	4	−·0843	−·0841	−	2			
„ H	+·0528	+·0526	+	2	−·0385	−·0381	−	4			
Mösting A	−·0906	−·0905	−	1	−·0557	−·0555	−	2	−·0900	−·0556	Franz
Triesnecker B	+·0068	+·0065	+	3	+·0205	+·0206	−	1	+·0066	+·0202	Pritchard
Murchison A	+·0198	+·0198		0	+·0697	+·0700	−	3	+·0174	+·0709	Neison
Pallas	−·0279	−·0288	+	9	+·0948	+·0954	−	6			
Bode...	−·0422	−·0420	−	2	+·1173	+·1177	−	4	−·0436	+·1155	Lohrmann
									−·0460	+·1155	Neison
Bode A	−·0200	−·0201	+	1	+·1568	+·1571	−	3	−·0229	+·1547	Neison

* To the E. of Airy, almost in contact with the wall.

	ξ_2	ξ_3	$\xi_2-\xi_3$	η_2	η_3	$\eta_2-\eta_3$	Other Observers. ξ	Other Observers. η	Other Observers. Authority.
Conon	+·0325	+·0324	+ 1	+·3699	+·3697	+ 2	+·0317	+·3669	Lohrmann
Aratus	+·0728	+·0726	+ 2	+·4006	+·4007	− 1			
Archimedes A	−·0982	−·0984	+ 2	+·4700	+·4703	− 3	−·1106	+·4656	Neison
„ *c*	−·0220	−·0225	+ 5	+·5241	+·5241	0			
„ *b*	−·0266	−·0278	+ 12	+·5705	+·5701	+ 4			
Piton	−·0102	−·0108	+ 6	+·6540	+·6528	+ 12			
Cassini *b*	+·0523	+·0517	+ 6	+·6428	+·6427	+ 1			
„ sm. crater* ...	+·0495	+·0488	+ 7	+·6605	+·6610	− 5			
Piazzi Smyth	−·0417	−·0421	+ 4	+·6674	+·6676	− 2			

The point measured has been the summit of the central mountain where one exists, in other cases the centre of the ring.

Out of the eighty-four differences only five exceed ·001, *i.e.* amount to 1″ (geocentric), twelve more exceed ·0005, the other sixty-seven are all less than half a second of arc.

The test is a severe one, as the points are measured under opposite illuminations, and very different librations. The effect of this is shown in the two columns of residuals, those for ξ, which is more affected by these causes, being considerably greater than those for η. But, if we bear in mind the great uncertainties which have been found in all previous determinations, I think the agreement must be considered most satisfactory.

Table II. gives the corresponding results for those points which were measured on both plates, but were not included in the determination of the orientation. Here, too, the agreement is hardly less satisfactory, though the average discordance in ξ is a little greater, as might have been anticipated, when it is remembered that one cause for rejection was unsuitable illumination.

* Just outside Cassini to the N.E.

TABLE II.

List of Twenty Points measured on Plates II. and III., but not used in Determining the Orientation.

	ξ_2	ξ_3	$\xi_2-\xi_3$	η_2	η_3	$\eta_2-\eta_3$	Other Observers. ξ	Other Observers. η	Other Observers. Authority
Maginus A*	−·0632	−·0641	+·0009	−·7635	−·7640	+·0005	−·0795	−·7655	Mädler
Licetus, small crater†	+·0232	+·c222	+ 10	−·7286	−·7288	+ 2			
Maurolycus A ...	+·1787	+·1773	+ 14	−·6877	−·6877	0	+·1719	−·6869	Mädler
Stöfler *f*	+·0643	+·0633	+ 10	−·6778	−·6774	− 4			
Walter	+·0139	+·0139	0	−·5469	−·5465	− 4			
Thebit A	−·0787	−·0790	+ 3	−·3677	−·3672	− 5	−·0939	−·3631	Mädler
Arzachael	−·0359	−·0359	0	−·3136	−·3140	+ 4			
„ A	−·0247	−·0247	0	−·3093	−·3092	− 1			
Alpetragius B ...	−·1150	−·1151	+ 1	−·2604	−·2605	+ 1			
Albategnius	+·0652	+·c650	+ 2	−·1958	−·1962	+ 4	+·0679	−·1969	Lohrmann
Halley	+·1000	+·0996	+ 4	−·1390	−·1393	+ 3	+·0970	−·1411	Neison
Hipparchus C ...	+·1414	+·1427	− 13	−·1281	−·1283	+ 2	+·1390	−·1285	Neison
„ L ...	+·1561	+·1566	− 5	−·1188	−·1191	+ 3	+·1523	−·1189	Neison
Horrocks	+·1023	+·1021	+ 2	−·0688	−·0694	+ 6	+·1023	−·0699	Neison
Bode B	−·0530	−·0530	0	+·1522	+·1523	− 1	−·0545	+·1515	Neison
Manilius	+·1540	+·1535	+ 5	+·2499	+·2498	+ 1	+·1479	+·2495	Bouvard and Nicollet
							+·1528	+·2463	Lohrmann
							+·1520	+·2515	T. Mayer
Aristillus S.E. peak ...	+·0173	+·0163	+ 10	+·5547	+·5548	− 1	+·0147	+·5557	Lohrmann
„ N.W. peak ...	+·0199	+·0187	+ 12	+·5572	+·5569	+ 3			
Kirsch	−·0751	−·0758	+ 7	+·6324	+·6328	− 4			
Cassini A	+·0636	+·0635	+ 1	+·6491	+·6495	− 4	+·0551	+·6478	Mädler.

* The central mountain, not the crater.

† A small round crater W. of *f* and *d*

§15. *Comparison with the Results obtained by previous Observers.*

Some of these positions differ considerably from those found by other observers. I may notice especially Maginus A and Thebit A in Table II.

In order to ensure that there has been no mistake with regard to the latter I have looked up the references to it in *Der Mond*. There is a misprint in the index which refers to it as Thebit B, but the description in the text leaves no doubt as to the identity. Its position was measured twelve times; the mean longitude derived from the first nine measures is $-5^\circ\ 37'$. I suppress the seconds, which are worthless, and the mean from the last three $-6^\circ\ 28'$. These last three were taken on the same night, and three other formations were also measured three times each on that night. Three out of the twelve measures were rejected because the positions deduced from them were so discordant as to show there must have been some error in the observations, whilst, with one exception only, each of the other nine gave a position further east than any single measure made on any other night. Mädler's two positions for Thebit A are in rectangular coordinates

From the first 9 measures $\xi = -\cdot0913$ $\eta = -\cdot3605$
From the last 3 measures $\xi = -\cdot1046$ $\eta = -\cdot3706$

showing a geocentric difference of $12''\cdot4$ in ξ and $9''\cdot4$ in η. This affords a pretty conclusive illustration of the errors to which measurements from the limb are liable.

But perhaps the most surprising result in Table II. is the large discordance in the position of Manilius, this having been considered as exceptionally well determined from 174 measures of Bouvard and Nicollet, made in their investigation of the Moon's physical libration.

I have not found this an easy point to measure, either on the photographs or on the Moon; but the two photographic measures are in close agreement with one another and with the telescopic measures quoted in § 6 (see also Table IV. below), and they are quite irreconcilable with Bouvard and Nicollet's position. It may be noted that while my latitude agrees with Bouvard and Nicollet's my longitude does not differ much from Lohrmann's. His, however, rests on only one determination. T. Mayer measured the point twenty-seven times, and his place is on the whole the nearest to mine.

In addition to the points measured on two plates I have measured sixty other points of the first order on one plate only, some on Plate II., others on III. I do not quote all these measures now, because the object of this paper is not to give definite results, but to call attention to the method and to the promise it holds out. It will, however, be interesting to compare the general results with those of previous observers.

TABLE III.

Comparison of the mean of all points measured with measures of previous Observers.

			Ratio.*	Mean Difference
Sum of 16 positive values of ξ measured by Lohrmann	4·8516	Sum of ξ's of same points from photo. measures	4·8727	1·0043	·0013
„ 21 negative „ „ „ „ „	6·2676	„ „ „ „ „ „ „	6·2629	·9993	·0002
„ 16 positive values of η „ „ „	5·6264	„ η „ „ „ „ „	5·6589	1·0058	·0020
„ 21 negative „ „ „ „ „	6·1058	„ „ „ „ „ „ „	6·1442	1·0063	·0018
Sum of 12 positive values of ξ measured by Mädler	3·8167	Sum of ξ's „ „ „ „ „	3·9079	1·0239	·0076
„ 28 negative „ „ „ „ „	10·0913	„ „ „ „ „ „ „	10·0784	·9987	·0005
„ 9 positive values of η „ „ „	4·7341	„ η „ „ „ „ „	4·7654	1·0066	·0035
„ 31 negative „ „ „ „ „	13·2423	„ „ „ „ „ „ „	13·3194	1·0058	·0025
Sum of 11 positive values of ξ measured by Neison	2·6022	Sum of ξ's „ „ „ „ „	2·6635	1·0236	·0056
„ 14 negative „ „ „ „ „	3·6882	„ „ „ „ „ „ „	3·6850	·9991	·0002
„ 13 positive values of η „ „ „	2·4759	„ η „ „ „ „ „	2·4959	1·0081	·0015
„ 12 negative „ „ „ „ „	1·9119	„ „ „ „ „ „ „	1·8910	·9890	·0017

* The value given by the photographic measures is always taken as the numerator of this ratio.

It will be seen that my results agree more closely with those of Lohrmann than with those of either Mädler or Neison. On the whole, the new measures place the points further from the centre than the old ones, a result which was anticipated in § 2. With regard to the curious difference noticeable in all three cases between the results for the positive and those for the negative values of ξ, I am not prepared to accept the obvious explanation of an error of centre in the photographic measures for reasons which will be stated in § 17.

It is only with regard to Mädler's measures that I have been able to investigate the details; but I find that, of his 28 points with negative ξ, 6 were measured from the west limb entirely; and another, Pico, was measured five times from each limb.

An examination of the few cases in which Mädler has measured a point sometimes from the east limb and sometimes from the west shows that his tendency is to place the point too far from the limb used. Thus in the case of Pico the five measures from the east limb give a longitude $-9^\circ\ 7'$, with $\xi=-\cdot1110$; those from the west limb give $-9^\circ\ 18'$, with $\xi=-\cdot1135$, the difference being ·0025, or $2''\cdot3$ (geocentric).

If these seven points are excluded from the table the numbers become, for 21 points:—

Sum of Mädler's values of ξ	...	...	9·3400
Sum of photographic values of ξ	...	...	9·3979
Giving the ratio	...	1·0062	
And mean difference	...	·0028	

which are in close accordance with the results obtained from the values of η.

This effect may be due to the difference in illumination, a rising sun tending to throw the point measured towards the east; but the matter requires further investigation.

It seems likely that a similar tendency to measure from the west limb rather than the east would be found among other observers. The former measures can be made in our evening, whilst the latter necessitate work in the early morning.

In no case did Mädler systematically measure a point in the west hemisphere from the east limb.

§ 16. *Argument that Absolute Places have really been found.*

The great differences already noted call for some further discussion of the accuracy of my results.

The close agreement between Plates II. and III. shown by Tables I. and II. proves, I think conclusively, that the relative positions are well determined. I will consider a little more closely how far the absolute places can be relied on.

Equations (10) in § 14 may be written

$$\left.\begin{aligned} 9478(\delta\theta_2 - \delta\theta_3) + 779\delta\theta_3 + 41{\cdot}50 = 0 \\ -8699(\delta\theta_2 - \delta\theta_3) + 710\delta\theta_3 - 42{\cdot}81 = 0 \end{aligned}\right\}$$

from which it appears that the coefficient of $\delta\theta_2 - \delta\theta_3$ is twelve times that of $\delta\theta_3$, and consequently that the value of $\delta\theta_2 - \delta\theta_3$ is much better determined than are those of $\delta\theta_2$, $\delta\theta_3$ separately; whilst it is upon these latter that the absolute places depend.

But if we take the value $\delta\theta_2 - \delta\theta_3 = -{\cdot}00465$, found from these equations, and substitute it in the four equations (8), (9), we have

From (8)	$334\delta\theta_3 = 1{\cdot}10$	whence	$\delta\theta_3 = {\cdot}00304$
	$176\delta\theta_3 = {\cdot}90$		$\delta\theta_3 = {\cdot}0051$
From (9)	$445\delta\theta_3 = 1{\cdot}47$		$\delta\theta_3 = {\cdot}00330$
	$886\delta\theta_3 = 3{\cdot}26$		$\delta\theta_3 = {\cdot}00368$

The only one of these which differs sensibly from the final solution $\delta\theta_3 = {\cdot}00331$ is that which depends upon the smallest coefficient.

Now equations (8) are the normal equations obtained from the conditional equations in ξ; equations (9) are similarly deduced from those in η; and as the values of A, E in equations (7), § 11, are nearly unity, whilst those of the other constants are small, it follows that the values of ξ depend chiefly upon the measures of x, those of η upon the measures of y, and therefore the two groups are to a large extent independent. The agreement between the values of $\delta\theta_3$ from these two groups shows that we are finding a real value, and not merely a chance mean.

It may be noted also that the probable error of $\delta\theta_3$ from the complete solution is only $0''{\cdot}9$.

These considerations seem to me to justify the conclusion that the orientation has been obtained absolutely within close limits.

§ 17. *Final Consideration of the Accuracy attained.*

The results in Table III. also raise the question whether the scale and centre have been accurately obtained for the photographic plates.

To test this I give in Table IV. a list of points measured on the Moon itself as explained in § 5, and also on the photographs. To my own measures I have again added, for ready comparison, those of other observers.

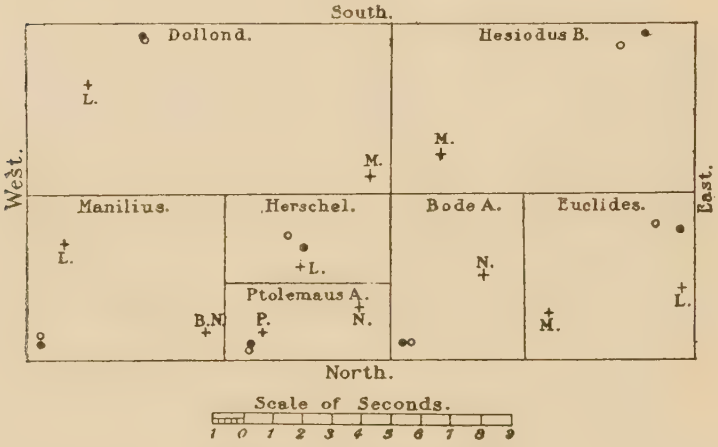
The scales in these two cases are entirely independent, that for the measures made at the telescope depending on the value adopted for my micrometer screw, which was determined from a series of measures of differences of declination of stars in the *Pleiades*, compared with the results of Elkin's triangulation.

TABLE IV.

Comparison of Points measured on the Photographs and at the Telescope.

	From Photographs.		At the Telescope.		By other Observers.		
	ξ	η	ξ	η	ξ	η	
Hesiodus B	−·2681	−·4557	−·2672	−·4553	−·2608	−·4515	Mädler
Dollond	+·2459	−·1818	+·2458	−·1817	+·2477	−·1801	Lohrmann
					+·2377	−·1768	Mädler
Ptolemaus A	−·0140	−·1479	−·0139	−·1477	−·0144	−·1483	Pritchard
					−·0168	−·1492	Neison
Euclides	−·4896	−·1279	−·4887	−·1281	−·4896	−·1258	Lohrmann
					−·4850	−·1249	Mädler
Herschel	−·0375	−·0986	−·0370	−·0990	−·0374	−·0979	Lohrmann
Bode A*	−·0200	+·1570	−·0203	+·1570	−·0229	+·1547	Neison
Manilius	+·1537	+·2499	+·1537	+·2496	+·1479	+·2495	Bouvard and Nicollet
					+·1528	+·2463	Lohrmann
					+·1520	+·2515	T. Mayer.

These results are also shown in the following diagram :—



● Position found from Photograph.
o „ „ at Telescope.
\+ „ „ by other Observers.

B.N.—Bouvard and Nicollet. L.—Lohrmann.
M.—Mädler. N.—Neison. P.—Pritchard.

The sum of the positive values of η is from the photographs 1·0119
„ „ „ at the telescope 1·0118
The sum of the negative values of η is from the photographs 4·069
„ „ „ at the telescope 4·066

* Measured on one night only.

So far as can be inferred from such a small number of measures this justifies the conclusion that both the scale and the ordinate of the centre have been correctly found.

But there is a mean difference of ·0004 between the negative values of ξ, the photographic values being numerically the greater. This, in conjunction with the fact that the ξ of Mösting A given in Table I. is —·0906, whilst Franz's value is —·0900, points to the conclusion that the centre has been placed about half a second too far west in the photographs. It will be remembered that it was noted in § 13 that some difficulty had been found in determining this abscissa.

Moving the centre to the east would numerically decrease the negative values of ξ, and accentuate the difference noted in Table III. between the results for positive and negative values. It is on this ground that I hesitate to accept an error in my position of the centre as an explanation of that difference.

With regard to this error of centre, it would have been possible to introduce corresponding terms into the conditional equations of § 14, but this would have necessitated the solution of eighty-four equations for six unknowns. Something of this sort may be desirable in the final work, but I did not feel that it was necessary in a preliminary investigation, which, for all I knew at the time, might end in a complete failure.

§ 18. *Conclusion.*

I do not think that the limit of accuracy of which these photographic measures are capable has yet been attained. In the first place, by measuring the original negatives I hope to find less difficulty in seeing the *réseau*, and to get better definition of some points which in the copies are lost in glare. Secondly, by the use of a *réseau* covering the whole limb, I hope to get a better determination of the coordinates of the centre and of the radius. Professor Turner has already procured for me a copy of one of sufficient size through the kindness of Mr. Franklin Adams.

M. Loewy has sent four more negatives, two of which show the zone already measured under yet different conditions. These I propose to treat in the same way as Plates II. and III. have already been dealt with, and so to obtain a set of definite values for the points shown on all four plates. When this has been done the constants for these plates can be finally determined, and the positions of all points shown on them obtained, with accuracy of the same order as those already measured.

This will greatly facilitate the determination of the constants for other plates, and of the positions of all points shown on them.

When once these constants have been found, the reduction of the measures is very simple, as may be seen from the following example :—

Reduction of Measures of Pitiscus A made on Plate II.

	x	y
Measured values of coordinates in terms of *réseau* intervals	10 835	24·360
Coordinates of centre of disc	17·345	13·752
Coordinates referred to centre of disc	+ 6·510	−10·808
Refraction correction (as in § 12)	2	3
Coordinates corrected for refraction	+6 512	−10·811
Multiply by ·063432 the reciprocal of Moon's radius.		
Coordinates in terms of Moon's radius	+·4131	− ·6858
Approximate value of $r = ·801$		
" " $z = ·599$		
Value of sin $s' = ·00440$.		
$\therefore z \sin s' = ·00264$.		
$r^2 \sin^2 s' = ·00002$.		
Correction for finite distance (as in § 8)	11	18
Final values of coordinates	+·4120	−·6840

Corresponding value of $z = +·6020$

The standard coordinates are found from

$$\xi = Ax + By + Cz.$$
$$= ·9786 \times (+·4120) + ·1863 \times (-·6840) + ·0876 \times (+·6020).$$
$$= +·3285.$$
$$\eta = Dx + Ey + Fz.$$
$$= -·1840 \times (+·4120) + ·9823 \times (-·6840) - ·0341 \times (+·6020).$$
$$= -·7682.$$

The measures have been made with one of Professor Turner's star plate micrometers belonging to the Oxford University Observatory. The glass scale is admirably adapted for work on the Moon, where it very frequently happens that one side of a crater is lost in glare. If the crater is circular and near the centre of the disc, the scale has only to be placed so that the readings of the points at which three of the arms cut the walls are equal, when the intersection must be at the centre of the circle. Other cases are not all so simple, but the help afforded by reading the points at which the arms cut the walls is always considerable. A crater has not been measured unless the greater part of the ridge is shown, the rest can be supplied mentally, and by determining the centre from this the confusing effects of light and shade upon the floor are avoided.

It only remains for me to express my thanks in the first place to M. Loewy for his great kindness in sending the photographs. The beauty of the negatives now taken at the Paris Observatory is too well known to need any comment, and it is little better than a truism to say that it is only upon photographs of the best definition that accurate measures are possible.

That I have been able to accomplish as much as I have in the time at my disposal has been due to the skill and the care with which Mr. Bellamy has overcome for me all the difficulties of

manipulation ; whilst the extent of my obligation to Professor Turner is very far indeed from being covered by the definite acknowledgments I have already made.

Some Suggestions for the Explicit Use of Direction Cosines or Rectangular Coordinates in Astronomical Computations. By H. H. Turner, M.A., F.R.S., Savilian Professor.

1. Until the introduction of the photographic method, the simple computations of astronomy could be fairly regarded as settled in form. The form depended essentially on the large use of meridian observations, which determine R.A. and N.P.D. independently, and thus all computations and catalogues are arranged in terms of R.A. and N.P.D. Tables of the planets, which are most conveniently expressed in latitude and longitude, and are so expressed in the first instance, are converted into terms of R.A. and N.P.D. for the use of observers.

2. But even independently of photography some inconveniences attending the use of these particular coordinates have made themselves felt. The neighbourhood of the pole is always a difficulty, and calls for some modification of the ordinary processes. Fabritius proposed in *Ast. Nach.* Nos. 2072–3 to use the coordinates

$$x=\sin p\cos\alpha \qquad y=\sin p\sin\alpha$$

in the neighbourhood of the pole, where α, p are the R.A. and N.P.D. of a star ; and he showed that the use of these rectangular coordinates considerably simplifies the calculation of precession and nutation for polar stars. His x and y may be thus regarded. Draw a tangent plane to the sphere at the pole, and orthogonally project stars from the sphere on to this plane by lines parallel to the Earth's axis ; then (x, y) are the Cartesian coordinates of the projection referred to axes in this plane.

3. In connection with the measurement and reduction of photographic plates I have urged the advantages of using "standard coordinates" which are also the Cartesian coordinates of a star supposed projected on a tangent plane to the celestial sphere ; but in this case the projection is not orthogonal ; it is by lines radiating from the centre of the sphere. For a plate with centre at the pole the "standard coordinates" would be

$$\xi=\tan p\cos\alpha \qquad \eta=\tan p\sin\alpha$$

as compared with Fabritius's

$$x=\sin p\cos\alpha \qquad y=\sin p\sin\alpha$$

4. Other projections on the tangent plane might be made. Thus the well-known stereographic projection would be represented by

$$X=2\tan\frac{p}{2}\cos\alpha \qquad Y=2\tan\frac{p}{2}\sin\alpha$$

and it is possible that these coordinates or yet others may be found useful in some department of astronomical work. But without following generalities further in this direction, we may notice that both Fabritius's and standard coordinates are closely related to direction cosines. If the three direction cosines of a star be

$$l=\sin p\cos\alpha \qquad m=\sin p\sin\alpha \qquad n=\cos p$$

then we have for Fabritius coordinates

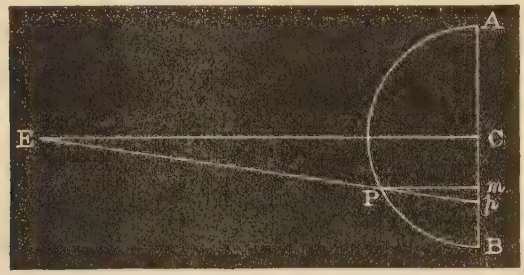
$$x=l \qquad y=m$$

and for standard coordinates

$$\xi=\frac{l}{n} \qquad \eta=\frac{m}{n}$$

Now this suggests that the more explicit use of direction cosines in our computations may be attended with advantages; and the present paper is written with the object of calling attention to this point by two particular examples.

5. The first example is that in which we are dealing with



positions of objects on the surface of the Sun or Moon. Let C be the centre of the Moon; E, the spectator on the Earth. He sees any point P on the surface of the Moon projected on the plane AB, as at p. Now E is at a great distance, and hence EP is *nearly* parallel, though not quite, to EC. By a slight correction to Cp we can find the orthogonally projected coordinates of P, and are thus prepared for the use of direction cosines in the reductions. The correction required is

$$\begin{aligned} pm &= \mathrm{P}m\tan m\mathrm{P}p \\ &= \sqrt{\mathrm{CB}^2-\mathrm{C}m^2}\,.\,\mathrm{C}p\,.\,/\mathrm{CE} \\ &= \frac{\mathrm{CB}}{\mathrm{CE}}\left\{1-\left(\frac{\mathrm{C}m}{\mathrm{CB}}\right)^2\right\}^{\frac{1}{2}}\mathrm{C}p \end{aligned}$$

Here CB/CE is the angular radius of the Moon in circular measure—about ·005. The remainder of the expression has a maximum value when $\mathrm{C}m^2=\frac{1}{2}\mathrm{CB}^2$, nearly; and thus the maximum value of the correction is

$$\tfrac{1}{2}\times\cdot005\times16'=2''\cdot5, \text{ say.}$$

Hence a small table suffices to give the fraction by which we must diminish C*p*.

If, then, we measure on the Moon, or on a photograph of the Moon, rectangular coordinates x' and y' from the centre of the disc, and expressed in terms of the Moon's radius, and with $\sqrt{x'^2+y'^2}$ as argument, take out from the table the appropriate reducing factor, we can get x, y, two of the direction cosines of the point, and can then find $z \equiv \sqrt{1-x^2-y^2}$.

6. These (x, y, z) are coordinates of the point with axis of z directed to the centre of the Earth and axes of x and y at right angles to it, with some orientation depending upon the way in which the measures were made. Let now (X, Y, Z) be direction cosines of the point referred to chosen axes fixed in the Moon. Then by the properties of direction cosines we know that

$$\xi = a_1x + a_2y + a_3z$$
$$\eta = b_1x + b_2y + b_3z$$
$$\zeta = c_1x + c_2y + c_3z$$

with the reciprocal relations

$$x = a_1\xi + b_1\eta + c_1\zeta$$
$$y = a_2\xi + b_2\eta + c_2\zeta$$
$$z = a_3\xi + b_3\eta + b_3\zeta$$

where $(a_1, a_2, a_3 \ldots c_3)$ are a system of direction cosines expressing the relationship of the two systems. We may calculate these coefficients from our knowledge of the Moon's librations ; or, if we know the coordinates (X, Y, Z) of certain points on the Moon, of which we determine the (x, y, z) by measures, we can find the coefficients $(a_1 \ldots c_3)$ by substituting for these known points in the equations above given.

7. Again, our measures may only give us x and y, affected with certain errors—those due to refraction, or defective centring of the plate, or defective scale value, or orientation. We shall thus get, not x and y, but two linear functions of them and z, viz.

$$x_0 = Ax + By + Cz + D$$
$$y_0 = Ex + Fy + Gz + H$$

where the coefficients AB . . . H are not altogether known, though certain parts of them, such as that due to refraction, may be calculated. Without troubling to make this calculation, however, we see from the linearity of all the above relations that we may write

$$x_0 = a\xi + b\eta + c\zeta + d$$
$$y_0 = e\xi + f\eta + g\zeta + h$$

so that the measures x_0 and y_0 (corrected to orthogonal projection of course) are expressible as linear functions of the coordinates (ξ, η, ζ). If, then, we know the (ξ, η, ζ) of a certain number of measured points we can find these eight constants of the plate $(a, b \ldots h)$, and then using them for any other measured point we can find the (ξ, η, ζ) from the two equations of the above form combined with (ξ, η, ζ).

8. Of this method Mr. Saunder (see p. 184) has made trial in measuring some beautiful lunar photographs, kindly lent by the Director of the Paris Observatory (M. Loewy). He found the method unsatisfactory, however, because there are, in fact, at present not enough points on the Moon whose positions are well determined. Mr. Saunder has therefore devised (see p. 185) an elegant method of his own, which bids fair to give us excellent positions of a number of points; and then it is possible that he may return to this method for finding the coordinates of other points.

9. But there is a point of detail in which the method requires improvement, viz. it is rather troublesome to solve the three equations

$$a\xi + b\eta + c\zeta = x_0 - d = x_1, \text{ say}$$
$$e\xi + f\eta + g\zeta = y_0 - h = y_1, \text{ say}$$
$$\xi^2 + \eta^2 + \zeta^2 = 1$$

(of which the third is a quadratic) for every point. The constants d and h are quickly subtracted, of course, and may be disregarded. To avoid the solution of the quadratic I suggest the following procedure:—

But for refraction and accidental errors the constants $a, b \ldots g$ would fulfil the relations

$$a^2 + b^2 + c^2 = 1,\ e^2 + f^2 + g^2 = 1,\ ae + bf + cg = 0$$

and then putting

$$1 - a^2 - e^2 = p^2,\ 1 - b^2 - f^2 = q^2,\ 1 - c^2 - g^2 = r^2,\ 1 - x_1^2 - y_1^2 = z_1^2$$

we should have

$$\xi = ax_1 + ey_1 + pz_1$$
$$\eta = bx_1 + fy_1 + qz_1$$
$$\zeta = cx_1 + gy_1 + rz_1$$

All these calculations could be quickly made, especially with an arithmometer and a table of squares.

Owing to refraction and accidental errors these relations are only approximately fulfilled. But if instead of x_1 and y_1 we write

$$x_1 + \lambda x_1 + \mu y_1 \text{ and } y_1 + \mu x_1 + \nu y_1,$$

where λ, μ, ν are small, we can determine (λ, μ, ν) so as to fulfil the corresponding relations *exactly*. We have, in fact,

$$(a+a\lambda+e\mu)^2+(b+b\lambda+f\mu)^2+(c+c\lambda+g\mu)^2=1$$
$$(e+a\mu+e\nu)^2+\quad .\quad .\quad .\quad .\quad .\quad =1$$
$$(a+a\lambda+e\mu)(e+a\mu+e\nu)+\quad .\quad .\quad .\quad =0$$

Put $1-(a^2+b^2+c^2)=A$, $1-(e^2+f^2+g^2)=C$, $ae+bf+cg=-B$,

where A, B, C are at least of the same order as λ, μ, ν; and neglect in the first instance squares of small quantities. The equations become

$$2\lambda=A,\ 2\nu=C,\ 2\mu=B,$$

and the test whether this approximation is sufficient is that the corrected values of the coefficients should sensibly fulfil the above relations. If not, the process may be repeated, and new values of λ, μ, ν found. But since refraction is small, one approximation should suffice. Putting, then,

$$\alpha=a+a\lambda+e\mu \quad \beta=b+b\lambda+f\mu \text{ \&c.}$$

and $\quad x_2=x_1+\lambda x_1+\mu y_1,\ y_2=y_1+\mu x_1+\nu y_1,\ {z_2}^2=1-{x_2}^2-{y_2}^2,$

so that α, β and also x_2, y_2, z_2 can be found without much trouble, we have

$$\xi=\alpha x_2+\epsilon y_2+\pi z_2$$
$$\eta=\beta x_2+\zeta y_2+\kappa z_2$$
$$\zeta=\gamma x_2+\eta y_2+\rho z_2$$

and thus ξ, η, ζ can be found simply.

10. A second example of the explicit use of linear formulæ of this kind is afforded by the computations of the effect of precession and nutation. The advantages have been partially pointed out by Fabritius and others. In *Ast. Nach.*, No. 3610, recently published, Dr. W. Ebert considers the special application of Fabritius's formulæ to stars within 20′ of the pole. But I venture to think that the advantages of the *general* application of such formulæ have been hitherto overlooked. Precession and nutation simply change the three axes of reference, so that if (x, y, z) are the direction cosines of a star for one date and (ξ, η, ζ) those for another, we have

$$\xi=a_1x+a_2y+a_3z$$
$$\eta=b_1x+b_2y+b_3z$$
$$\zeta=c_1x+c_2y+c_3z$$

where the constants a_1, a_2, a_3 . . . c_3 are connected by the six well-known relations. Since the axes of reference change slowly

a_1, b_2, c_3 are nearly unity, and the others nearly zero, for any moderate interval. We may write, in fact,

$$\xi - x = ax + a_2 y + a_3 z$$
$$\eta - y = b_1 x + \beta y + b_3 z$$
$$\zeta - z = c_1 x + c_2 y + \gamma z$$

and then the coefficients on the right are all small, and only approximate values of x, y, z are needed on the right.

11. Some of the coefficients are of the first order and some of the second. To show their general character let us neglect the slow motion of the ecliptic and consider the pole of the equator (axis of z or ζ) revolving round the fixed pole of the ecliptic with angular velocity q. Let x, y, z be the coordinates of a star at epoch $t=0$, and ξ, η, ζ the coordinates at time $t=t$.

The pole of the ecliptic has the same coordinates in both systems, viz.

$$x=0 \qquad y=-\sin\omega \qquad z=\cos\omega$$
$$\xi=0 \qquad \eta=-\sin\omega \qquad \zeta=\cos\omega$$

Thus

$$0=-a_2\sin\omega+a_3\cos\omega$$
$$-\sin\omega=-b_2\sin\omega+b_3\cos\omega$$
$$\cos\omega=-c_2\sin\omega+c_3\cos\omega.$$

Therefore

$$\tan\omega=\frac{a_3}{a_2}=\frac{b_3}{b_2-1}=\frac{c_3-1}{c_2}.$$

Multiply the numerators and denominators by a_3, b_3, c_3 respectively and add. Each ratio is found equal to $\frac{1-c_3}{-b_3}$ and this shows that $b_3=c_2$. The same result is obtained if we multiply by a_2, b_2, c_2. But from multiplying by a_1, b_1, c_1 we get each ratio equal to $\frac{c_1}{b_1}$; whence $c_1=b_1\tan\omega$. It will be found that we can now express all the quantities in terms of c_3. Thus

$$c_2 = (c_3-1)\cot\omega$$
$${c_1}^2 = 1-{c_2}^2-{c_3}^2$$
$$b_1 = c_1\cot\omega$$
$$b_3 = (c_3-1)\cot\omega$$
$${b^2}_2 = 1-{b_1}^2-{b_3}^2$$

and so on. To find c_3 we must refer to the spherical triangle formed by E, the pole of the ecliptic, and P_1 P_2, the two poles of the equator at the different epochs. In this triangle

$$c_3=\cos P_1P_2=\cos P_1E\cos P_2E+\sin P_1E\sin P_2E\cos E$$
$$=\cos^2\omega+\sin^2\omega\,\mathrm{ccs}\,qt.$$

Hence $c_2 = \sin\omega\cos\omega(\cos qt - 1)$

$c_1 = \sin\omega\sin qt$

$b_1 = \cos\omega\sin qt.$

We may express the values by the following scheme :—

	x	y	z
ξ	$\cos qt$	$-\cos\omega\sin qt$	$-\sin\omega\sin qt$
η	$\cos\omega\sin qt$	$\cos^2\omega\cos qt + \sin^2\omega$	$\cos\omega\sin\omega(\cos qt - 1)$
ζ	$\sin\omega\sin qt$	$\cos\omega\sin\omega(\cos qt - 1)$	$\sin^2\omega\cos qt + \cos^2\omega.$

12. Now qt is a small quantity. In a century $qt = 5000''$ approximately, or, say, ·025 in circular measure. Thus we may exhibit the approximate values of powers of qt for different periods as follows :—

	10 yrs.	50 yrs.	100 yrs.	300 yrs.
qt	·0025	·0125	·025	·075
$(qt)^2$	·0000063	·0001563	·000625	·005625
$(qt)^3$	·0000000	·0000020	·0000156	·0004188
$(qt)^4$	...	·0000000	·0000004	·0000316
$(qt)^5$	...	...	·0000000	·0000024

Hence, if we are applying precessions for ten years, we can neglect the third power of qt; for fifty years the fourth power; and, indeed, up to one hundred years we can neglect the fourth power, which, as we shall see, is generally multiplied by a small fraction.

13. For simplicity let us first neglect $(qt)^3$—*i.e.* consider precessions for moderate periods comparable with ten or twenty years. Then putting $qt\cos\omega = s$, $\tan\omega = r$; since $\cos\omega = ·91$, s is less than qt, and its powers converge more rapidly. Also powers of $r(=0·4)$ converge quickly. The scheme of transformation now becomes

	x	y	z
ξ	$1 - \frac{1}{2}(1+r^2)s^2$	$-s$	$-rs$
η	$+s$	$1 - \frac{1}{2}s^2$	$-\frac{1}{2}rs^2$
ζ	$+rs$	$-\frac{1}{2}rs^2$	$1 - \frac{1}{2}r^2s^2$

Since s is proportional to the first power of the time, and s^2 to the second, we see that this scheme gives in a compendious form both the precessions and secular variations of x, y, z—viz. the precessions are

in x, $\quad -(y + rz)s$

in y, $\quad +x \,.\, s$

in z, $\quad +xr \,.\, s$

and the secular variations are

$$\begin{aligned} \text{in } x \quad & -\tfrac{1}{2}x(1+r^2)s^2 \\ \text{in } y \quad & -\tfrac{1}{2}(y+rz)s^2 \\ \text{in } z \quad & -\tfrac{1}{2}r(y+rz)s^2 \end{aligned}$$

14. Similarly, the additional terms of the third and fourth orders are readily written down as follows, omitting a common factor, $\frac{1}{24}s^3(1+r^2)$, for convenience in writing :—

	x	y	z
ξ	$s(1+r^2)$	4	$4r$
η	-4	$+s$	$+sr$
ζ	$-4r$	$+sr$	$+s\,.\,r^2$

The terms of the third order are all less than $\frac{1}{6}(qt)^3$, and only begin to affect the seventh place of decimals after about thirty years.

The largest term of the fourth order is $\frac{1}{24}(qt)^4$, and does not affect the seventh place for more than a century.

15. Since the ecliptic is slowly moving, terms must be added to these coefficients, depending on the motion of the ecliptic ; but these will be of a higher order still ; and enough has been said to show the general character of the coefficients. Returning to the general form for them

$$\begin{aligned} \xi &= a_1x + a_2y + a_3z \\ \eta &= b_1x + b_2y + b_3z \\ \zeta &= c_1x + c_2y + c_3z \end{aligned}$$

Suppose (x, y, z) refer to any standard epoch—say 1900·0. Then, to find the coordinates for any other epoch, we want the nine coefficients, a_1, a_2 . . . c_3. Now it would not be a difficult matter to tabulate these for every year for three hundred years—say, from A.D. 1700 to 2000 (all we are likely to want at present)—and we should then have the means of bringing up to 1900·0 *accurately* any stellar positions referred to another epoch. There is no special difficulty about polar stars ; the formulæ never become less simple in any part of the celestial sphere ; and the labour of multiplication can be quickly performed with an arithmometer, or in many cases by Crelle's Tables, since the coefficients are small.

16. To reduce from one epoch to another when neither is 1900·0, we must form nine new coefficients ; but these are readily formed when we have those connecting both epochs with 1900·0. Thus let (ξ, η, ζ) refer to one epoch, as above, and (X, Y, Z) to another, and let

$$\mathrm{X} = \mathrm{A}_1x + \mathrm{A}_2y + \mathrm{A}_3z, \text{ \&c.}$$

so that

$$x = \mathrm{A}_1\mathrm{X} + \mathrm{B}_1\mathrm{Y} + \mathrm{C}_1\mathrm{Z}, \text{ \&c.}$$

according to the usual inversion of direction cosines.

Then $\xi = a_1x + a_2y + a_3z$

$$= (a_1A_1 + a_2A_2 + a_3A_3)X + (a_1B_1 + a_2B_2 + a_3B_3)Y + (a_1C_1 + a_2C_2 + a_3C_3)Z$$

with similar equations for η and ζ, and the law of formation of the coefficients is obvious.

17. *Aberration.*—The use of direction cosines does not always simplify the formulæ, however, and we may take aberration as an instance where there is no great simplification. The effect of aberration is well known to be precisely similar to that of parallax, viz. a displacement of the observer in a given direction. If the direction be defined by the cosines (l, m, n), and k be the coefficient of aberration, the centre of the sphere is virtually moved to the point $(-kl, -km, -kn)$. Thus the coordinates of a point (x, y, z) become

$$p(x + kl),\ p(y + km),\ p(z + kn),$$

p being a constant introduced in order that the sum of the squares may remain unity ; *i.e.*

$$p^2 + 2p^2k(ln + my + nz) + p^2k^2 = 1$$

Now $k = 20''{\cdot}5$ and $k^2 = 0''{\cdot}002$, which is insensible in practice.

Thus neglecting k^2, we have

$$p = 1 - k \cos \theta$$

where $\cos \theta = lx + my + nz$, so that θ is the "Earth's Way," the angle between the direction of the star and that of the Earth's motion ; and to the first order of k we have for the increments of x, y, z due to aberration

$$k(l - x \cos \theta),\ k(m - y \cos \theta),\ k(n - z \cos \theta).$$

Thus the increment of any coordinate x due to precession, nutation, and aberration, is

$$kl + a_1x + a_2y + a_3z - kx(lx + my + nz),$$

and as regards the calculation of star-corrections, we do not appear to gain anything by the use of direction cosines.

18. It may be remarked incidentally that the effect of aberration on "standard coordinates"

$$\xi = \frac{x}{z} \text{ and } \eta = \frac{y}{z}$$

is to change them into

$$\xi^1 = \frac{x + kl}{z + kn} \text{ and } \eta^1 = \frac{y + km}{z + kn},$$

the common factor p disappearing in the division and leaving only the constants (for all stars) kl, km, kn. When we are dealing with a small photographic plate for which z is nearly unity, we may put $z+kn=z(1+kn)$, and $kl/(z+kn)=kl$,
and thus $\xi'=\xi(1-kn)+kl$.

There is thus a displacement of the centre by the quantities (kl, km) and a change of scale, ξ and η being both multiplied by $(1-kn)$. This result is derived in another way in *Monthly Notices*, vol. liv. p. 20.

Summary.

The present paper points out the advantages of using direction cosines or rectangular coordinates in certain astronomical computations, instead of curvilinear coordinates.

(*a*). In mapping the surface of the Sun or Moon, observations can be easily corrected so as to give us an orthogonal projection of the surface on a diametral plane : which is the same as giving two of three direction cosines. The third can be easily found, and then the transformation to any other axis is made by linear formulæ.

(*b*). In applying precession for long periods to star places, the formulæ in terms of direction cosines are very simple, and a small amount of tabulation would render the accurate reduction of one catalogue to another a simple matter.

The Extra-Equatorial Currents of Jupiter in 1899. By Rev. T. E. R. Phillips.

Despite the southern declination of *Jupiter* in 1899, an immense amount of detail was visible on the planet's surface, and numerous observations have been received from several observers, which have enabled the rotation periods of the surface material in various latitudes to be determined with, it is believed, considerable accuracy. The writer secured a large number of transits of spots during the earlier and intermediate months of the apparition. Mr. Denning, most fortunately, was able to pursue his observations very late—in some cases up to the middle of September; Mr. A. S. Williams has forwarded a large and valuable series of transits; and, in addition to these, much assistance has been derived from the figures of Messrs. Gledhill, Antoniadi, and J. Comas Solà, of Català, Spain (the observations of the latter being published in *Astronomische Nachrichten*, No. 3596, Band 150). Many of the spots have thus been followed for considerable periods of time (in some cases for more than seven months), and in the majority of cases the observations are so numerous and so accordant as to make the question of identi-

fication quite certain. It may be well to state here that those spots of which the observations admit of any reasonable doubt have been carefully excluded from this discussion.*

The writer wishes to draw special attention to the altogether abnormal velocity exhibited by several spots on the N. tropical zone.

The rotation periods have been obtained in nearly every case by comparing a few observations at the beginning of the series with a few at the end, and the adopted value for each spot has been tested by working out the residuals for all the observations obtained.

All the longitudes in this paper are calculated according to System II. of Mr. Crommelin's ephemeris.

The zero meridian of this system has a daily rate of $870°{\cdot}27$, which corresponds to a rotation period of $9^h\ 55^m\ 40^s{\cdot}63$.

The observations are so numerous that it is impossible to give them in detail. They are therefore (with the exception of those relating to the red spot) presented in chart form.

These charts are so arranged as to show the observed longitudes of the spots, the dates of the observations, and the observers by whom the transits were obtained.

The following is the explanation of the symbols employed :—

Dark spot.	White spot.	Observer.
▮	‖	Antoniadi.
▲	△	Denning.
◆	◇	Gledhill.
●	○	Phillips.
▼	▽	Solà.
■	□	Williams.

Though necessarily wanting in the accuracy of figures, it is hoped that the charts will have the advantage of enabling the reader

(1) To judge of the correctness or otherwise of the identifications ;

(2) To see at a glance to what extent in each case the adopted period (indicated by the straight line drawn through the observations) really represents the mean motion of the spot ; and

(3) To note readily the various fluctuations of velocity which many of the spots undoubtedly exhibit.

In the tables in which the spots are summarised, and in the second column, the letters D and W denote *dark* and *white* spots respectively.

* See, however, discussion of S. tropical spots.

The following abbreviations are also employed in this discussion :—

N.N.T.B.=north north temperate belt.
N.T.B.=north temperate belt.
S.T.B.=south temperate belt.
S.S.T.B.=south south temperate belt.
N.E.B.=north equatorial belt.
R=rotation period.
p=preceding.
f=following.

1. *Northern Spots.* Approx. lat. +24° to +33°.

Included under this head are spots which appeared on the N.N.T.B. and, later in the apparition, on the N.T.B. The former of these belts was fairly conspicuous throughout the apparition, and almost invariably seen double. On the other hand, the N.T.B. was for some months practically invisible. Towards the middle of April the writer traced it as an exceedingly feeble line from about λ 145° to about λ 290°, where it appeared to merge into the N.N.T.B. Later still a few dusky streaks made their appearance, but being ill-defined in character their observation was a matter of some difficulty.

Several transits of spots in about lat. +25° have come to hand, but their identifications are in nearly every case so doubtful that it has been thought better to omit them rather than introduce an element of uncertainty into the results. In consequence only one spot on the N.T.B. has been included in this discussion.

The spots on the N.N.T.B. (lat. +30°±) were more numerous and more pronounced, though their positions on the disc, combined with the tilt of the N. hemisphere away from the Earth and the planet's considerable S. declination, caused much difficulty in accurately determining the time of their central meridian transits. It is therefore only to be expected that the observations should show some discordances.

The dark spots frequently appeared double, both components of the belt being intensified and united by dusky shadings, which, according to Mr. A. S. Williams, projected as faint streaks beyond the belt in a N. and S. direction.

In a few places the writer observed distinct white spots, which either indented the N.N.T.B. on its S. edge or formed complete rifts through the belt. These white spots seemed to require good seeing conditions to be satisfactorily dealt with, and only one was sufficiently well observed to be included in this paper.

In 1898 two dark streaks on the N.N.T.B. gave a mean rate for this latitude of $9^h\ 55^m\ 52^s{\cdot}0$ (*Monthly Notices*, 1898 December). In the present year the mean derived from four spots for the zone comprised within about lat. +24° and lat. +33° was

$$9^h\ \ 55^m\ \ 50^s{\cdot}6.$$

Chart I.

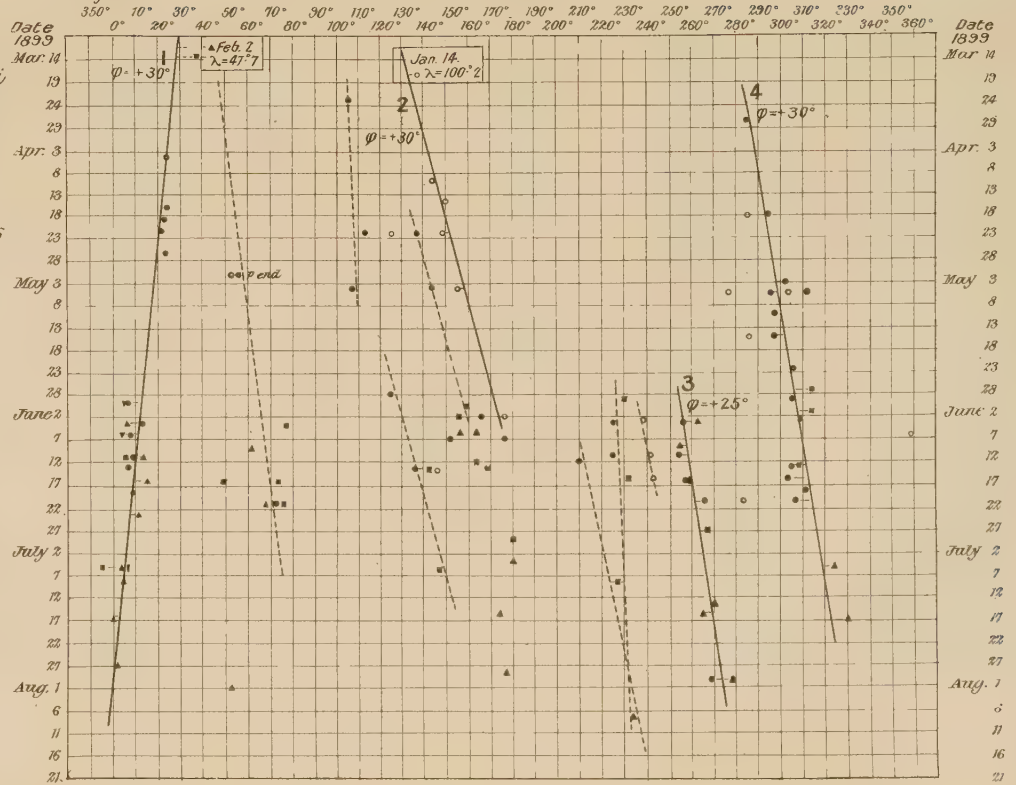
SPOTS ON N.T.B AND N.N.T.B.

Lat +24°to+33° Mean R=9h 55m 56·9s

derived from spots 2,3 & 4.

Dark spot	White spot	Observer
I	II	*Antoniadi*
▲	△	*Denning*
◆	◇	*Gledhill*
●	○	*Phillips*
▼	▽	*Sola*
■	□	*Williams*

Period adopted for Longitudes
9h 55m 40s·63



The spots, however, did not move at anything like a uniform rate. A white spot which formed a complete break in the N.N.T.B. indicated a period of $9^h\ 56^m\ 1^s{\cdot}5$, which was longer than the period of any other spot of which the identification was beyond doubt. On the other hand there was a remarkable spot on the N.N.T.B., which, between the beginning of April and the end of July, exhibited the abnormally short period for this latitude of

$$9^h\ \ 55^m\ \ 32^s{\cdot}0.$$

It is probable that this rapidly moving spot was quite an isolated case. It therefore seems better in giving the average value for this latitude to take the mean of the three spots which are fairly accordant, and appear to represent the true motion of the surface material in this region of the planet.

The figures then are

$$R=9^h\ \ 55^m\ \ 56^s{\cdot}9.$$

Table of Spots (*see* Chart I., Plate 5).

No.	Character of Spot.	Period of Observation.	No. of Observations.	Rotation Period.	Remarks.
		Days		h m s	
1	D	114	24*	9 55 32·0	On N.N.T.B. Small and well defined at first, more diffused later.
	W	139	6	9 56 1·5	Break through N.N.T.B. Well seen in good air.
3	D	58	12	9 55 54·2	On N.N.T.B.
4	D	112	18	9 55 54·9	Diffused condensation of N.N.T.B.

The approximate lat. (ϕ) of the spots discussed is marked on the chart.

2. *The North Tropical Zone.* Approx. lat. $+11°$ to $+18°$.

Beyond all question this was the most remarkable region of the planet in 1899. It will be remembered that several dark spots were seen on this zone in 1898, strung like beads on a very narrow line a little to the N. of the north equatorial belt.

* There is good reason for believing that the following two transits also refer to this spot:

1899 Feb. 2 at $19^h\ 9^m$ $\lambda=47{\cdot}7°$ (observed by Mr. Denning), and
„ Mar. 13 at $16^h\ 3^m$ $\lambda=38{\cdot}3°$ (observed by Mr. Williams).

Comparing Mr. Denning's transit of February 2 with that of July 27, we get a mean period of

$$9^h\ \ 55^m\ \ 29^s{\cdot}8.$$

As, however, this period does not accord so well with that of the rest of the observations, and the two transits above mentioned stand as it were alone, and may possibly be identified differently (see Chart I.), it is, perhaps, better to compute the velocity of the spot from the transits secured between the beginning of April and the end of July as above.

Three of these spots were sufficiently dark and conspicuous to be thoroughly well observed, and from these Mr. Denning found a mean rotation period of $9^h\ 55^m\ 26^s{\cdot}3$. When the planet was observed by the writer at the end of 1898 December, it was at once noticed that in certain longitudes the north equatorial belt (which had become markedly broader) was much disturbed and broken up along its N. edge by a series of light and dark spots. It appeared as if the dark material of the belt had extended so far N. as to join in many places the narrow line on which the dark spots of the previous apparition had been seen. On subsequent occasions, however, when the seeing was really good, this line was distinctly visible, but the N. edge of the great N. belt had undoubtedly advanced, as the N. tropical spots appeared sometimes (in poor seeing) in contact with, and at other times distinctly separated from, the belt. By the middle of March the observations secured were sufficiently numerous to show that the majority of the spots were moving at a distinctly slower rate than in 1898. But a surprising fact was soon observed. Attention was called by the writer in the *English Mechanic* (No. 1780, letter 42,347) to a dark N. tropical spot which showed an altogether abnormal rate of motion. Later on, other spots made their appearance which exhibited a similar rapidity of movement, and on working out the final results it was found that whereas the surface material in one region of the zone was rotating at a mean rate of $9^h\ 55^m\ 33^s{\cdot}9$ (which is about normal for this latitude), the other part was rotating in $9^h\ 55^m\ 15^s{\cdot}3$. Differences of velocity in different longitudes are of course not infrequent, but it is most remarkable that any N. tropical spots should move with such rapidity as those to which reference has just been made ; in fact, such a rate of motion has never—to the writer's knowledge—been recorded of any spots in this latitude before.

The sharpness of the boundary line between these two regions was no less striking, for the rapidly moving material appeared to be strictly confined within about λ 140° and λ 260°. Spots starting immediately W. of this latter point showed a remarkable velocity from their commencement, and spots on approaching λ 140° were either quenched and died out, or they were suddenly checked and made to conform to the slower rate of the spots preceding them.

A large number of spots in this zone were sufficiently well observed to enable reliable periods to be worked out.

Mean R of all the spots discussed$=9^h\ 55^m\ 28^s{\cdot}0$.

Mean of 17 spots between λ 260° and λ 140° (approximately)

$9^h\ 55^m\ 33^s{\cdot}9$.

Mean of 8 spots between λ 140° and λ 260°

$9^h\ 55^m\ 15^s{\cdot}3$.

Chart II.

N. TROPICAL SPOTS.

Lat. +11° to +18° Mean R = 9h 55m 28·0s

Dark spot	White spot	Observer
I	II	Antoniadi
▲	△	Denning
◆	◇	Gledhill
●	○	Phillips
▼	▽	Sola
■	□	Williams

Period adopted for Longitudes
9h 55m 40s·63

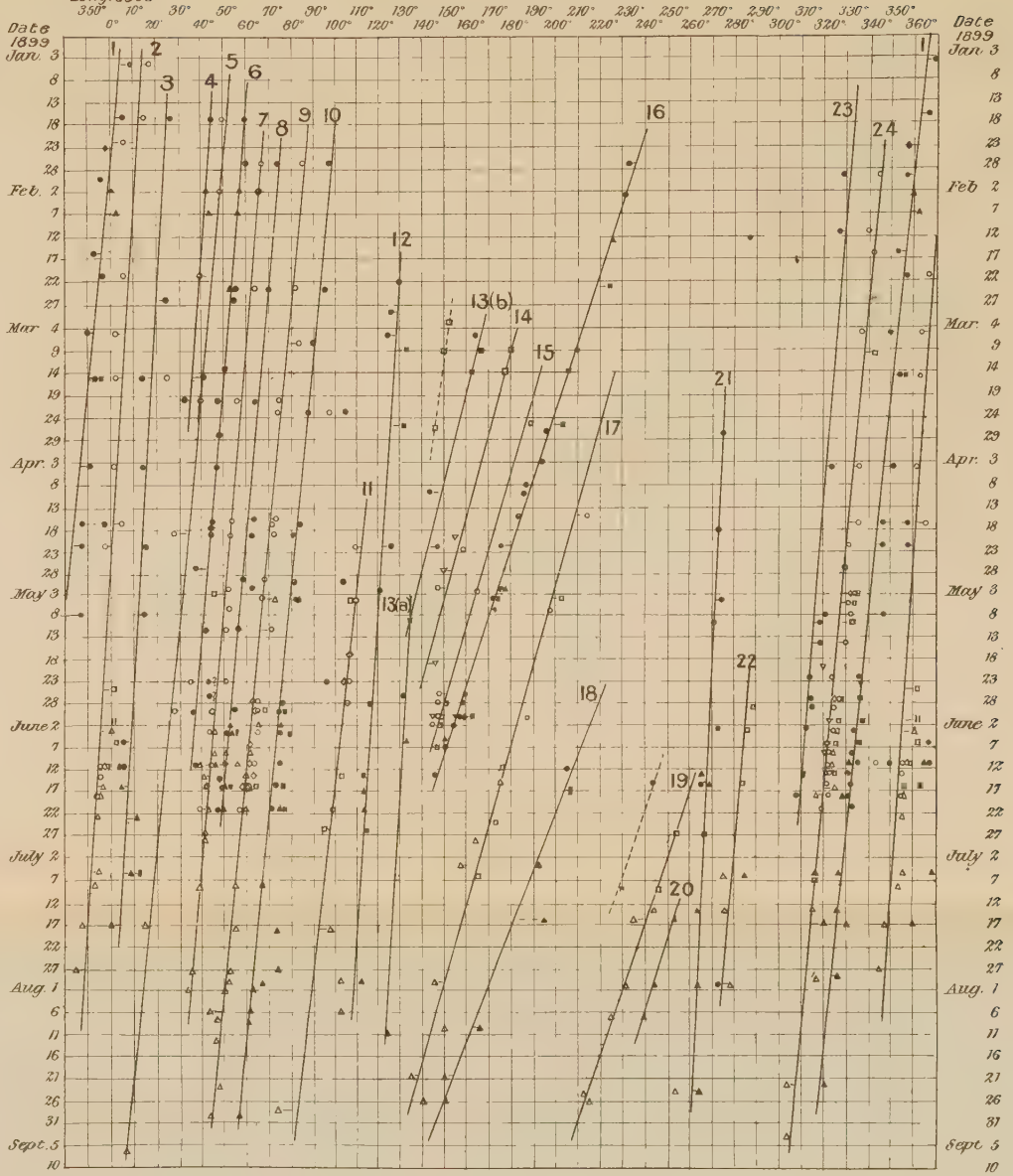


Table of Spots (*see* Chart II., Plate 6).

No.	Character of Spot.	Period of Observation. Days	No. of Observations.	Rotation Period. h m s	Remarks.
1	D	230	33	9 55 32·5	
2	W	204	25	9 55 35·7	Motion irregular. Spot intensely brilliant on June 11.
3	D	182	14	9 55 35·9	Spot rather small and faint.
4	D	63	5	9 55 35·8	Spot small and faint.
5	W	233	8	9 55 32·7	Spot faint and observations somewhat discordant.
6	D	146	20	9 55 33·7	Darkest marking on the planet, but died out in June. Quite detached from N.E.B. in good seeing. Probably identical with spot "A" of 1898.
7	W	187	27	9 55 33·4	
8	D	146	22	9 55 33·0	
9	W	216	41	9 55 32·5	Very bright and conspicuous.
10	D	216	27	9 55 32·7	Rather long spot. Detached from N.E.B.
11	W	128	12	9 55 31·4	Bright and rather large.
12	D	158	13	9 55 35·2	Long and dark.
13*a*	D	94	4	9 55 35·7	Really one spot, but sudden change of velocity near beginning of May.
13*b*	D	64	6	9 55 20·1	
14	W	71	7	9 55 18·2	Velocity checked on approaching λ 140°.
15	W	73	11	9 55 15·8	It is possible that this and spot 14 united at end of May to form one large spot.
16	D	138	28	9 55 14·2	Well-defined dark spot.
17	W	133	14	9 55 17·1	Velocity more rapid at first, but becoming slackened later.
18	D	75	7	9 55 8·5	Velocity most abnormal.
19	W	60	8	9 55 12·2	Moderately large and bright.
20	D	22	3	9 55 16·3	Period of observation very short, but identity certain, and velocity accords well with that of other spots.
21	D	150	12	9 55 36·5	Very long dark spot detached from N.E.B.
22	W	63	6	9 55 33·1	
23	D	139	12	9 55 34·5	Motion not uniform.
24	W	216	50	9 55 32·8	Remarkably brilliant and definite oval spot.

3. *The Red Spot.* Approx. lat.—21°.

This spot was again exceedingly faint and difficult, but, as was the case in 1898, with the exception of its S. edge, where it was in contact with the S.T.B., the complete oval outline could just be detected when the atmospheric conditions were favourable. To the writer the spot again appeared grey ; but Mr. Williams speaks of it as exhibiting at times a feeble reddish or pinkish glow. At its *f* end the spot was fairly distinct, but the *p* end was almost of the last degree of faintness, and completely obliterated in imperfect seeing. It was, therefore, generally thought preferable to observe the central meridian transits of the well-known "hollow" or "bay" in the south equatorial belt, in which the red spot lies, rather than those of the spot itself. The former object was easily observed, even under the worst seeing conditions, and its rotation period may safely be regarded as identical with that of the spot.

With regard to the position of the spot there is some difference of opinion. The majority of observers appear to agree in placing it somewhat nearer the *f* end of the "hollow," and to the writer its central meridian passages seemed as much as three minutes later than those of the centre of the latter object. On the other hand, Mr. A. S. Williams states that he saw the spot of such a length that its centre was rather inclined to the *p* side of the "hollow."* His transits, which refer to the passages of the spot itself, are consequently considerably earlier than those of other observers.

Between the end of 1898 July and the close of the last apparition the "red-spot hollow" exhibited a striking irregularity of motion. On the planet's reappearance in the morning sky it was found from observations made in 1899 January that the hollow was about 3° or 4° ahead of its computed position according to its velocity of 1898. For the first few months of the apparition the increased velocity thus indicated was maintained, the rotation period at this time being not very much greater than that of the zero meridian of System II. (Mr. Crommelin's ephemeris). From 1898 August to 1899 April Mr. Denning found a mean period of $9^h\ 55^m\ 41^s{\cdot}2$; but, taking the apparition of 1899 separately and as a whole, the following value seems to satisfactorily represent the observations :—

$$R=9^h\ 55^m\ 42^s{\cdot}0.$$

As the red spot is by far the most important and interesting feature on the surface of the Planet, the observations relating to this object are given, not in chart form, but in detail. The

* Mr. Williams wishes it to be understood that this statement refers to the centre of *figure* of the spot. He agrees with the other observers in putting the centre of *plainness* or *density* of the spot distinctly on the *f* side of the centre of the "hollow."

fourth column (O—C) contains the residuals or the differences between the observed positions and those calculated according to the adopted period of $9^h\ 55^m\ 42^s{\cdot}0$. The last column contains the initials of the observers, as follows :—

A=Antoniadi	P=Phillips
D=Denning	S=Solà
F=Flammarion	W=Williams
G=Gledhill	

The residuals in the following table refer to transits of the "hollow," the observations of the spot itself being denoted by an asterisk. The transits of J. Comas Solà relate to the relatively dark *f* end of the spot.

Date. 1899.	Time of C. M. passage. h m	λ (System II.) °	O−C °	Observer.
Jan. 4	19 37	27·3	−0·7	P.
4	19 40	29·1	*	P.
16	19 36	29·4	+1·0	P.
16	19 39	31·2	*	P.
Feb. 2	18 39	29·5	+0·5	D.
7	17 46	29·0	−0·1	D.
21	19 18	29·0	−0·6	P.
21	19 20	30·2	*	P.
24	16 49	30·0	+0·3	D.
26	18 27	29·9	+0·1	D.
26	18 28·5	30·8	+1·0	P.
26	18 31·5	32·6	*	P.
Mar. 13	15 44	26·8	*	W.
15	17 25	28·5	−1·8	P.
20	16 34	29·7	−0·8	P.
28	13 10	29 5	−1·3	P.
April 4	13 53·5	28·7	−2·3	P.
12	10 36	32·3	+1·0	G.
16	13 48	30·2	−1·2	P.
16	13 52	32·6	+1·2	G.
17	9 41	31·2	−0·2	G.
19	11 15	29 0	−2·5	P.
19	11 20	32·0	+0·5	D.
21	12 53·5	29·2	−2·4	P.
26	12 2·5	30·5	−1·2	P.
26	12 3	30·8	−0·9	D.
26	12 3	30 8	−0·9	G.
30	...	47·1	*	S.
May 1	11 10	30·7	−1·2	P.
3	12 44	28·3	*	W.

Date. 1899.	Time of C. M. passage.	λ (System II.)	O−C	Observer.
	h m	°	°	
May 4	8 44	33·6	+1·6	G.
6	10 15	29·3	*	W.
6	10 17	30·5	−1·5	P.
6	10 17	30·5	−1·5	MacEwen
6	10 19	31·7	−0·3	D.
6	10 20	32·3	+0·3	G.
8	11 54·5	30·2	−1·9	P.
8	11 57	31·7	−0·4	G.
8	11 58	32·3	+0·2	D.
11	9 24	30·4	−1·8	P.
16	8 34	31·9	−0·5	G.
18	10 14	33·0	+0·6	G.
23	9 21	32·7	+0·1	P.
25	11 0	33·1	+0·4	G.
25	11 0·3	33·3	*	W.
28	8 27	31·6	−1·2	P.
28	8 30	33·4	+0·6	G.
30	10 3	30·2	*	W.
30	10 8	33·2	+0·4	G.
30	10 9	33·8	+1·0	P.
30	10 9·5	34·1	+1·3	A.
June 4	9 16	33·2	+0·2	P.
4	9 17	33·8	+0·8	G.
4	9 17·5	34·1	*	W.
4	9 18	34·4	+1·4	D.
4	9 18	34·4	+1·4	F.
4	9 20·1	35·8	*	F.
6	10 46·5	28·4	*	W.
6	10 54·5	33·2	+0·1	P.
6	10 56	34·1	+1·0	G.
6	10 57	34·7	+1·6	D.
6	...	46·5	*	S.
9	8 25	33·8	+0·6	G.
9	8 26	34·4	+1·2	D.
9	...	52·0	*	S.
11	9 58·2	30·5	*	W.
11	10 2·5	33·1	−0·1	P.
11	10 3	33·4	+0·2	G.
11	10 4	34·0	+0·8	D.

Date. 1899.		Time of C. M. passage. h m	λ (System II.) °	O−C °	Observer.
June	14	7 32	32·9	−0·4	G.
	14	7 32	32·9	−0·4	D.
	16	9 8	31·4	*	W.
	16	9 12	33·8	+0·4	G.
	16	9 13	34·4	+1·0	D.
	16	9 14	35·0	+1·6	P.
	18	10 50·5	33·8	+0·3	P.
	21	8 20	33·5	−0·1	G.
	21	8 20	33·5	−0·1	D.
	23	9 58	33·1	−0·5	D.
	26	7 28	33·1	−0·6	G.
	26	7 29	33·7	0·0	D.
	28	9 8	33·9	+0·1	D.
July	5	9 56	34·1	+0·1	G.
	5	9 58	35·3	+1·3	D.
	5	...	45·3	*	S.
	8	7 27	34·6	+0·5	D.
	15	8 15	34·6	+0·2	G.
	17	9 56	36·0	+1·6	D.
	27	8 10	33·1	−1·7	G.
	27	8 11	33·7	−1·1	D.
	29	9 49	33·3	−1·5	D.
	30	5 47	37·0	+2·1	D.
Aug.	1	7 22	34·6	−0·3	P.
	1	7 22·5	34·9	0·0	D.
	6	6 29	33·1	−2·0	D.
	8	8 9	33·7	−1·5	D.
	8	8 11	34·9	−0·3	P.
	11	5 42	35·1	−0·2	D.
	13	7 22	35·7	+0·4	D.
	18	6 30	34·6	−0·9	D.
	23	5 44	37·0	+1·3	D.
	30	6 33	37·0	+1·1	D.
Sept.	6	7 21	36·4	+0·3	D.
	14	3 59	34·7	−1·7	D.
	16	5 41	36·5	0·0	D.

Note.—Mr. Denning secured a transit of the red-spot "hollow" on 1898 November 29 at $19^h\ 55^m$ ($\lambda = 31°{\cdot}9$). This observation appears to be somewhat late, but it is valuable as indicating that the acceleration of velocity referred to above probably did not set in till late in the autumn.

4. *South Tropical Spots.* Approx. lat. −18° to −28°

Mr. Williams has sent some valuable observations of white spots on the N. tropical zone, *i.e.* the bright zone between the S. edge of the south equatorial belt and the N. edge of the S.T.B. Four of these spots appear to give a mean value for this region of

$$9^h\ 55^m\ 20^s{\cdot}7.$$

The observations, however, are few in number, and might perhaps be identified differently. The above result, therefore, though probably correct, must necessarily be regarded as open to some degree of doubt. It is much to be regretted that no additional observations of these spots are available for discussion.

Table of Spots.

No.	Character of Spot.	Period of Observation. Days.	No. of Observations.	Rotation Period. h m s	Remarks.
1	W.	73	3	9 55 20·4	Very small and faint.
2	W.	24	3	9 55 26·8	Very small and faint.
3	W.	64	3	9 55 11·6	Very faint.
4	W.	60	3	9 55 23·8	Small and faint.

5. *South Temperate Spots.* Approx. lat. −28° to −35°.

A considerable number of dark and white spots were visible on or at the S. edge of the S.T.B. during the apparition of 1899. In 1898 the period derived from six spots was $9^h\ 55^m\ 19^s{\cdot}4$ (*Monthly Notices*, 1898 December). In the present year fifteen spots give a mean of

$$9^h\ 55^m\ 19^s{\cdot}3.$$

As is the case with most of the Jovian currents, the spots in this latitude differed considerably amongst themselves, and the velocities of the individual spots varied from time to time. A glance at the chart, however, will show that there is no room for doubt as to the question of identification, and the mean result may be relied upon as being very near the truth.

Some of the white spots projected so far into the white S. temp. zone that they appear to have been largely influenced by the more rapidly drifting material of the great southern current. The periods of the dark spots on the S.T.B. in about latitude −30° were far more uniform.

Chart III.

S. Temp. Spots

Lat.- 28° to -35° Mean R = 9h 55m 19·3s

Dark spot	White spot	Observer
I	II	Antoniadi
▲	△	Denning
◆	◇	Gledhill
●	○	Phillips
▼	▽	Sola
■	□	Williams

Period adopted for Longitudes
9h 55m 40s·63

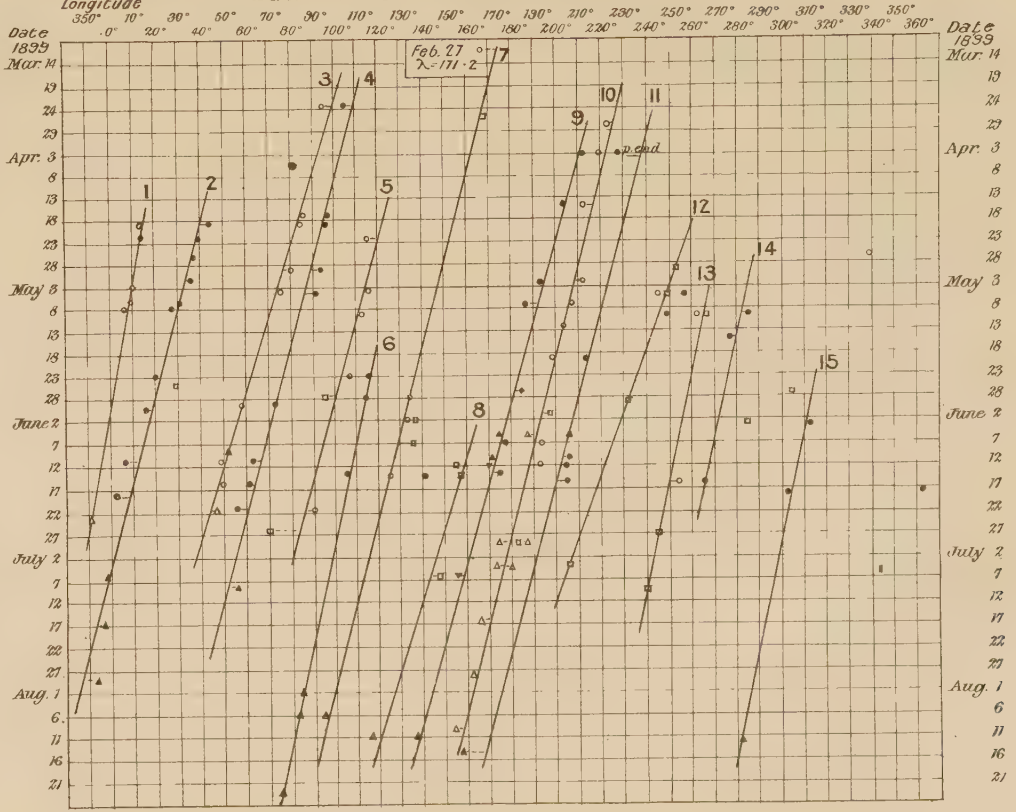


Chart IV.

SOUTHERN SPOTS

Lat. -36° to -50° Mean R = 9h 55m 7·0s

Dark spot	White spot	Observer
I	II	Antoniadi
▲	△	Denning
◆	◇	Gledhill
●	○	Phillips
▼	▽	Sola
■	□	Williams

Period adopted for Longitudes
9h 55m 40s·63

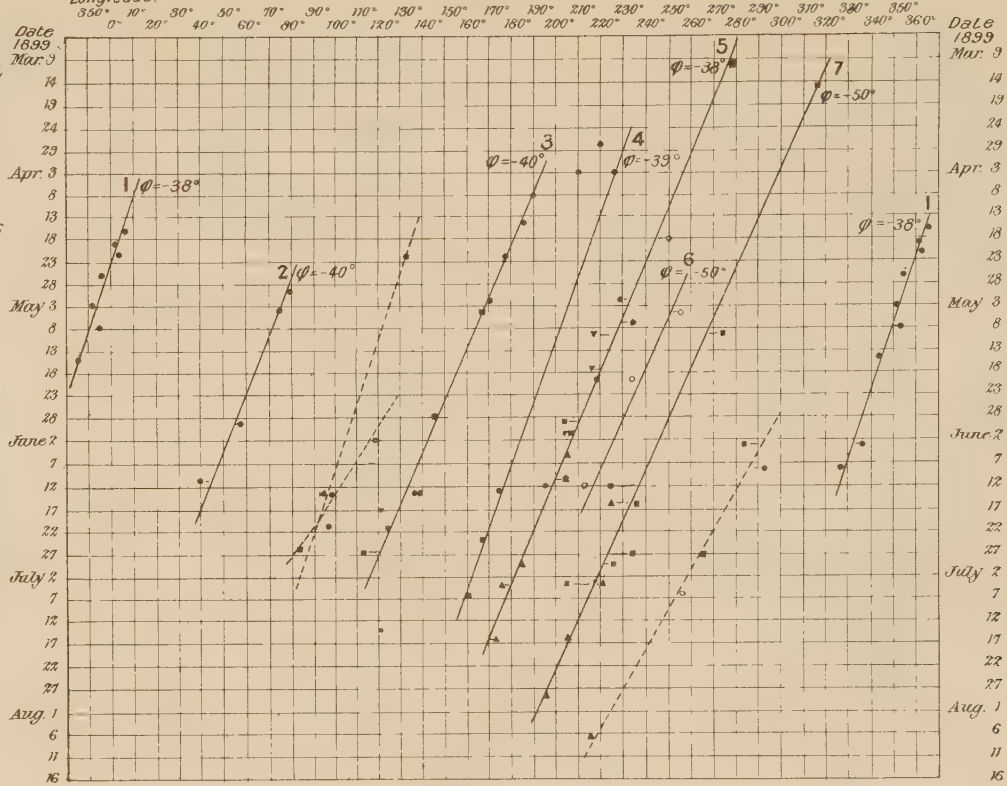


Table of Spots (*see* Chart III., Plate 7).

No.	Character of Spot.	Period of Observation. Days.	No. of Observations.	Rotation Period. h m s	Remarks.
1	W	65	6	9 55 24·4	
2	D	101	13	9 55 19·2	Rotation much more rapid till latter part of June, then suddenly slackened.
3	W	90	10	9 55 16·6	
4	D	107	10	9 55 19·0	Rather dark. Motion varied considerably.
5	W	65	7	9 55 17·5	Velocity varied considerably.
6	D	92	6	9 55 22·3	Observations remarkably accordant.
7	W	160	8	9 55 18·7	
8	W	60	4	9 55 13·8	
9	D	130	13	9 55 18·2	
10	W	140	19	9 55 19·7	Bright spot.
11	D	133	7	9 55 18·9	
12	W	65	5	9 55 11·7	
13	W	60	5	9 55 24·8	
14	D	35	3	9 55 20·5	
15	D	70	3	9 55 24·0	

6. *The Great Southern Current.* Approx. lat. $-36°$ to $-50°$.

This current was very well observed in 1899, the mean result being in close agreement with that of 1898. As is only to be expected, the observations, relating as they do to more or less faint markings in high latitudes, show some discordances ; but, on the whole, the agreement is very satisfactory and the indentifications beyond doubt.

The mean rotation period derived from the observations of seven spots works out as

$$9^h\ 55^m\ 7^s{\cdot}0,$$

which is just $0^s{\cdot}7$ greater than that derived from two spots in 1898.

Table of Spots (*see* Chart IV., Plate 8).

No.	Character of Spot.	Period of Observation.	No. of Observations.	Rotation Period. h m s	Remarks.
1	D	53	9	9 55 13·4	Dark spot at *p* end of dusky streak.
2	D	43	4	9 55 7·3	Following end of dark streak.
3	D	79	10	9 55 4·4	
4	D	94	4	9 55 11·7	Observations scanty, but no doubt as to identity.
5	D	128	17	9 55 6·2	
6	W	38	3	9 55 3·2	At mouth of rift in N. polar shading.
7	D	135	11	9 55 3·1	Observations rather discordant.

Note.—The approximate latitude (ϕ) of the spots is given on the chart.

Summary of Results.

	Current	Approx. Lat.	No of Spots.	Rotation Period. h	m	s
1	Northern spots	From +24° to +33°	1	9	55	32·0
			3	9	55	56·9
2	N. trop. spots	From +11° to +18°	25	9	55	28·0
			(17	9	55	33·9)
			8)	9	55	15·3
3	"Red spot"	−21°	1	9	55	42·0
4	S. trop. spots	From −18° to −28°	4	9	55	20·7 *
5	S. temp. spots	From −28° to −35°	15	9	55	19·3
6	Southern spots	From −36° to −50°	7	9	55	7·0

* This result must be regarded as somewhat doubtful.

Observations of the Lunar Eclipse, 1899 *December* 16, *at the Liverpool Observatory.* By W. E. Plummer.

The weather conditions at the time of the lunar eclipse were not very favourable for the observation of occultations. During the early part of the eclipse the sky was quite cloudy, the Moon being only occasionally visible through short breaks.

The following observations were made with the 8-inch equatorial and a power of 80. The times noted were thought to be generally satisfactory, but the identification of the stars is by no means certain.

Star.	Phase.	Greenwich Mean Time. h	m	s
B. D. 22° No. 1003	Disapp.	12	57	59·9
,, 22 ,, 1004	,,	13	7	5·7
,, 22 ,, 1006	,,	13	14	53·2
,, 22 ,, 1011	,,	13	20	24·8
,, 22 ,, 993	Reapp.	13	28	47·9
,, 22 ,, 1006	,,	13	48	43·4

The occultation of *Neptune* was looked for, but the planet was lost in the glare of the Moon as it approached the limb.

An attempt was made near the time of maximum eclipse to determine the angular breadth of the unobscured portion, but the observation proved impossible. The attempt seemed to indicate that the portion in the penumbra was greater than was predicted, but the uncertainty of the effects of irradiation and the difficulty of deciding on the termination of the shadow rendered any definite conclusion impossible.

Liverpool Observatory,
1900 *Jan.* 11.

Note on the Variable Star η Argûs. By Col. E. E. Markwick.

As the recorded observations of the magnitude of this historic variable star appear to have been but few and far between in past time, and as I made a pretty careful determination of it in the years 1883 and 1884, I think it worth while to record it.

The observations were made at Pietermaritzburg, Natal, with a $2\frac{3}{4}$-inch refractor, eyepiece, power about 28.

1883	Dec. 28	7·5	Two or three steps < D; about = B.
1884	Feb. 5	7·54	Between B and D in brightness.
	Feb. 18	7·6	While observing, a cloud came over, dimming η and the surrounding stars. I thus had a good opportunity of noting the different times of disappearance of the stars. After careful observation I came to the conclusion that it was about = D in brightness.
	June 19	7·46	Between stars A and B in brightness.
	20	7·46	As before.

The comparison stars used were :

	m.	
A = B 3202	7·44	As at p. 256, *Uran. Arg.*
B = B 3204	7·48	
D = G 1332	7·60	

There was little or no variation apparently while the star was under observation, so I think the net result of the above may be taken as

1884·23 7·51

My glass was then only provided with a very rough equatorial stand without circles, but I made certain of the identification of η on 1883 June 20 when I compared its position with that of star D. Using an eyepiece with three threads, one parallel to a circle of declination, and two each inclined 45° to the former, the telescope was adjusted so that D ran along the central thread. Then the times of D passing the centre and of η *Argûs* passing the two threads were noted eight times. A mean of these was taken, and with the corresponding differences in R.A. and Dec. the position of η was ascertained, that of D being taken from the *U. Arg.* The resulting position ($10^h\ 40^m\ 14^s$ R.A. 59° 1′·6 S. Dec.) was such as to ensure and fix at once the correct identification of η, differing as it did from the catalogue position by 1^s only in R.A. and 0′·1 in Dec. This procedure was very necessary, as the variable is surrounded by numbers of stars of about the same degree of brightness, from which by its appearance alone I found it at first impossible to distinguish it.

Moreover, to add to possible confusion, the circle indicating variability is placed round the wrong star in the U.A. Map, viz. No. 227 (U.A.) *Carinæ*, a star a little to the S. of the true variable. This error is, however, duly noted in the body of the work.

Combining my result with the data in Mr. Innes's Paper (*Monthly Notices*, vol. lix. p. 570) we get the following :—

1884·23	7·51
1886·2	7·60
1896·4	7·58
1897·2	7·60
1899·5	7·71

It would hence appear that this star, which once (1843) exceeded every star but *Sirius* in brightness, has, for the sixteen years 1883–1889 remained at or about $7^{m}\cdot6$ with a possible slight tendency to diminish in magnitude.

Observations of Meteors at the Royal Alfred Observatory, Mauritius, 1899 November.

(*Communicated by T. F. Claxton.*)

The Leonid display witnessed at Mauritius was but meagre, though the results may be of some value as indicating what occurred at this station, whose latitude is 20° 5′ 39″ S., and longitude $3^{h}\ 50^{m}\ 12^{s}\cdot6$ E.

Watch was kept by three observers on the mornings of November 14 and 15 (from midnight to sunrise), and on the 16th one observer watched at intervals, but saw no meteors.

It is probable that the maximum occurred after sunrise.

On November 14 the Moon set at 2.20 A.M., and at 3.5 A.M. on the 15th.

Until 2 A.M. on the 14th the sky in the neighbourhood of the radiant was occasionally covered with very thin misty clouds through which bright stars only were faintly visible, otherwise the weather on each night was fine and clear.

The zodiacal light was observed on the 14th and 15th. The axis of the cone at 2.30 on each morning appeared to extend from *α Leonis* along a parallel of declination to the horizon, the apex gradually receding from that star as the morning advanced, or, in other words, keeping at an approximately constant altitude. The base stretched for about 20° along the horizon. No pink tinge was distinguishable.

Mauritius (Civil Time).	Apparent Size.	Duration in Secs.	Colour.	Train.	From		To		Observer.
					R.A.	Dec.	R.A.	Dec.	
1899. h. m. s.	m	s			°	°	°	°	
Nov. 14 2 41 14	3	1	Yellow	None	120	+30	137	+28	C.
2 59 32	2	1	,,	Slight	128	+22	145	+21	C.
3 8 35	3	1	Bluish white	Broken, 3 secs.	124	+40	120	+44	C.
3 10 24	3	½	,,	None	147	+13	160	+15	O.
3 14 54	3	½	,,	,,	117	+42	107	+58	C.
3 23 53	4	1	,,	Slight	148	+23	147	+32	C.
3 25 0	3	1	,,	,,	115	+18	132	+15	O.
3 30 22	3	½	Yellow	,,	162	+25	175	+30	C.
h. m. s.									
Nov. 15 0 26 56	>1	2	Bluish	Red	104	+39	118	+51	C.
0 48 9	>1	3	Bluish white	Slight	123	+40	134	+39	C.
1 28 51	>1	2½	,,	,,	113	+ 5	144	+ 3	O.
2 28 18	2	½	,,	,,	128	+ 4	135	+11	C.
32 28	2	½	Bluish	Slight, ½ sec.	142	+11	149	+ 7	W.
33 14	1	½	Bluish white	None	71	+32	68	+43	C.
45 26	3	½	Yellow	Slight	150	−11	157	−33	C. & O.
54 31	2	1	Bluish white	Broken, 1 sec.	120	+27	103	+32	C. & W.
3 7 31	2	2	,,	Slight	163	−17	173	−17	C. & O.
12 37	2	1	Bluish	,,	150	−16	148	−31	W.
13 38	1	1	Bluish white	,,	166	+22	175	+21	C.
19 48	3	½	,,	,,	162	−17	173	−17	C.
25 54	1	½	,,	,,	170	− 2	182	− 8	C.
25 59	2	1	,,	,,	166	−22	166	−30	O.
30 23	3	½	Bluish	,,	152	− 9	165	−24	W.
32 33	3	1	Bluish white	,,	175	−17	186	−26	C.
34 42	2	1	,,	,,	180	−16	186	−24	W.
35 47	2	1	,,	,,	133	+47	120	+58	C.
36 22	2	1	Bluish	,,	133	+47	120	+58	O.
43 40	1	1	,,	Brilliant	167	−50	172	−61	W.
49 20	3	½	Bluish white	None	150	+18	166	+21	C.
49 30	3	1	,,	Train	150	+ 2	150	−11	W.
52 1	2	½	,,	Train, 1 sec.	160	+11	170	+ 4	C.
55 6	1	1	Bluish	Slight	150	+10	175	+15	O.
55 58	1	½	,,	Brilliant	185	−15	188	−19	W.
59 16	2	½	Bluish white	Bright	160	+ 9	170	− 2	C.
4 0 46	2	½	,,	Slight	166	+19	178	+15	W.
9 35	2	1½	,,	None	170	− 2	182	− 8	C.
9 43	1	1	,,	Brilliant	150	−45	143	−61	W.
10 44	1	1½	Bluish	Train, ½ sec.	170	− 2	182	− 8	O.
13 30	1	½	Bluish white	Fine	151	−12	126	−23	C.

The initials C., W., O. are those of Mr. Claxton, Mr. Walter, and Mr. Olivier respectively.

Observations of Occultations of Stars by the Moon and of Phenomena of Jupiter's Satellites made at the Royal Observatory, Greenwich, in the Year 1899.

Communicated by the Astronomer-Royal.

Day. 1899.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Jan. 19	Disapp. μ Arietis	Old Altazimuth	100	Dark	8 4 21·29	P. M.
Feb. 22	Disapp. Piazzi VIII. 106	Sheepshanks Equat.	100	,,	12 41 44·13	H.
22	,, ,,	Astrographic Equat.	120	,,	12 41 43·61	R. C.
22 (*a*)	Reapp. ,,	Sheepshanks Equat.	100	Bright	13 50 38·91	H.
April 17	Disapp. 3 Cancri	,, ,,	100	Dark	10 44 47·71	R.
May 26 (*a*)	Reapp. 7 Sagittarii	,, ,,	55	,,	12 38 53·37	W. B.
26	,, 9 Sagittarii	,, ,,	55	,,	13 15 24·89	W. B.
July 19	,, Bradley 2153	,, ,,	55	Bright	9 17 16·81	W. S.
19	,, 26 Ophiuchi	,, ,,	55	,,	9 24 38·11	W. S.
Dec. 16	Disapp. B. D. 22° 991	28-inch Equat.	200	Eclipsed	12 16 8·46	W. B.
16	,, Lal. 10708–9	Merz 13-inch Equat.	265	,,	12 17 2 66	D.
16 (*b*)	,, ,,	Astrographic Equat.	225	,,	12 17 2·50	H.
16	,, ,,	28-inch Equat.	200	,,	12 17 3·13	W. B.
16	,, ,,	Sheepshanks Equat.	150	,,	12 17 2·81	W.

Day. 1899.	Phenomenon.		Telescope.	Power.	Moon's Limb	Mean Solar Time of Observation. h m s	Observer.
Dec. 16	Disapp.	Piazzi V. 184	Merz 13-inch Equat.	265	Eclipsed	12 22 55·38	D.
16 (*b*)	,,	,,	Astrographic Equat.	225	,,	12 22 55·42	H.
16	,,	,,	28-inch Equat.	200	,,	12 22 56·15	W. B.
16	,,	,,	Sheepshanks Equat.	150	,,	12 22 55·94	W.
16	,,	B. D. 22° 999	Merz. 13-inch Equat.	265	,,	12 25 5·82	D.
16 (*b*)	,,	,,	Astrographic Equat.	225	,,	12 25 6·37	H.
16	,,	,,	28-inch Equat.	200	,,	12 25 6·71	W. B.
16 (*c*)	,,	,,	Sheepshanks Equat.	150	,,	(12 25 9·09)	W.
16 (*c*)	,,	B. D. 22° 998	28-inch Equat.	200	,,	12 27 44·18	W. B.
16 (*d*)	,,	B. D. 22° 1000	Merz 13-inch Equat.	265	,,	12 37 1·96	D.
16	,,	,,	Astrographic Equat.	225	,,	12 37 1·53	H.
16	,,	,,	28-inch Equat.	200	,,	12 37 1·85	W. B.
16	,,	,,	Sheepshanks Equat.	150	,,	12 37 1·95	W.
16	,,	Anonymous *b*	28-inch Equat.	200	,,	12 41 39·29	W. B.
16 (*c*)	,,	Lal. 10739–40	Merz 13-inch Equat.	265	,,	12 54 38·95	D.
16	,,	,,	Astrographic Equat.	225	,,	12 54 38·15	H.
16	,,	,,	28-inch Equat.	200	,,	12 54 39·06	W. B.
16	,,	,,	Sheepshanks Equat.	150	,,	12 54 39·38	W.

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1899.						
Dec. 16	Disapp. Lal. 10739–40	Corbett Tel.	100	Eclipsed	12 54 39·16	P. M.
16	,, Lal. 10741–2	Merz 13-inch Equat.	265	,,	13 0 53·83	D.
16	,, ,,	Astrographic Equat.	225	,,	13 0 54·04	H.
16	,, ,,	28-inch Equat.	200	,,	13 0 53·94	W. B.
16	,, ,,	Sheepshanks Equat.	150	,,	13 0 53·57	W.
16	,, ,,	Corbett Tel.	100	,,	13 0 53·84	P. M.
16	,, Anonymous *c*	28-inch Equat.	200	,,	13 3 53·35	W. B.
16	,, Anonymous *d*	,, ,,	200	,,	13 5 35·67	W. B.
16	,, Anonymous *e*	,, ,,	200	,,	13 6 37·01	W. B.
16	,, Anonymous *f*	,, ,,	200	,,	13 8 19·13	W. B.
16	,, Lal. 10745	Merz. 13-inch Equat.	265	,,	13 9 4·19	D.
16 (*b*)	,, ,,	Astrographic Equat.	225	,,	13 9 4·12	H.
16	,, ,,	28-inch Equat.	200	,,	13 9 4·61	W. B.
16	,, ,,	Sheepshanks Equat.	150	,,	13 9 4·66	W.
16	,, Anonymous *g*	28-inch Equat.	200	,,	13 14 3·58	W. B.
16	Reapp. B. D. 22° 991	Merz 13-inch Equat.	265	,,	13 20 34·89	D.
16 (*e*)	,, ,,	Astrographic Equat.	225	,,	13 20 34·73	H.
16	,, ,,	28-inch Equat.	200	,,	13 20 35·21	W. B.

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1899.						
Dec. 16 (*a*)	Reapp. B. D. 22° 991	Sheepshanks Equat.	150	Eclipsed	(13 20 51·08)	W.
16	Disapp. B. D. 22° 1009	Merz. 13-inch Equat.	265	,,	13 28 20·60	D.
16 (*a*)	,, ,,	Astrographic Equat.	225	,,	13 28 19·66	H.
16	,, ,,	28-inch Equat.	200	,,	13 28 19·44	W. B.
16 (*c*)	,, ,,	Sheepshanks Equat.	150	,,	(13 28 24·81)	W.
16	Reapp. Piazzi V. 184	Merz 13-inch Equat.	265	,,	13 31 2·16	D.
16 (*e*)	,, ,,	Astrographic Equat.	225	,,	13 31 2·22	H.
16	,, ,,	28-inch Equat.	200	,,	13 31 2·50	W. B.
16	,, ,,	Sheepshanks Equat.	150	,,	13 31 1·77	W.
16	,, Lal. 10708–9	Merz 13-inch Equat.	265	,,	13 35 25·45	D.
16 (*e*)	,, ,,	Astrographic Equat.	225	,,	13 35 25·92	H.
16	,, ,,	28-inch Equat.	200	,,	13 35 26·19	W. B.
16	,, ,,	Sheepshanks Equat.	150	,,	13 35 26·07	W.
16	,, B. D. 22° 1010	Merz 13-inch Equat.	265	,,	13 39 58·70	D.
16	,, ,,	Astrographic Equat.	225	,,	13 40 0·57	H.
16	,, ,,	28-inch Equat.	200	,,	13 39 59·44	W. B.
16	,, ,,	Sheepshanks Equat.	150	,,	(13 40 3·43)	W.
16	,, B. D. 22° 999	Merz 13 inch Equat.	265	,,	13 40 24·53	D.

Day. 1899.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Dec. 16	Reapp. B. D. 22° 999	28-inch Equat.	200	Eclipsed	13 40 25·27	W. B.
16	Disapp. B. D. 22° 1014	Merz 13-inch Equat.	265	,,	13 49 46·58	D.
16	,, ,,	28-inch Equat.	200	,,	13 49 46·33	W. B.
16	,, Anonymous *h*	,, ,,	200	,,	13 54 3·73	W. B.
16	,, Anonymous *j*	,, ,,	200	,,	13 55 55·43	W. B.
16	Reapp. Lal. 10745	Merz 13-inch Equat.	265	,,	13 57 36·80	D.
16	,, ,,	Astrographic Equat.	225	,,	13 57 33·80	H.
16	,, ,,	Sheepshanks Equat.	150	,,	13 57 36·27	W.
16	,, Lal. 10741–2	Merz 13-inch Equat.	265	,,	13 58 22·48	D.
16	,, ,,	Astrographic Equat.	225	,,	13 58 21·58	H.
16	,, B. D. 22° 1003	Merz 13-inch Equat.	265	,,	(14 10 40·66)	D.
16 (*a*)	,, ,,	Astrographic Equat.	225	,,	14 10 34·09	H.
16	,, ,,	28-inch Equat.	200	,,	14 10 34·43	W. B.
16	,, ,,	Sheepshanks Equat.	150	,,	14 10 32·56	W.
16	,, B. D. 22° 1009	28-inch Equat.	200	,,	14 31 0·68	W. B.

Notes.

(*a*) Not considered a good observation. (*b*) Star apparently projected on limb before disappearance.
(*c*) Star very faint. (*d*) Star appeared to take 0·2ˢ to disappear. (*e*) Sudden.

Phenomena of Jupiter's Satellites.

Day.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation. h m s	Mean Solar Time of *N.A.* h m	Observer.
1899. April 18	III. (*a*)	Occ. R. Last contact	Sheepshanks Equat.	120	10 51 4·91	10 41	B.
May 6	II.	Tr. Ing. First contact	,,	,,	9 40 3·99		H.
6	II.	Bisection	,,	,,	9 42 48·55	9 42	,,
6	II.	Last contact	,,	,,	9 45 23·13		,,
6	I.	Occ. D. First contact	,,	,,	11 53 57·17		,,
6	I.	Bisection	,,	,,	11 56 11·80	11 56	,,
6	I.	Last seen	,,	,,	11 58 21·45		,,
6	II.	Tr. Egr. First contact	,,	,,	11 55 11·97		,,
6	II.	Bisection	,,	,,	11 56 56·68	11 56	,,
6	II.	Last contact	,,	,,	12 0 41·06		,,

Note.

(*a*) The satellite emerged very slowly.

The observations of occultations of stars on December 16 were made during a partial eclipse of the Moon.

The initials D., H., B., W. B., R., H. F., R. C., W., P. M., W. S., are those of Mr. Dyson, Mr. Hollis, Mr. Bryant, Mr. Bowyer, Mr. Rendell, Mr. Furner, Mr. Cheeseman, Mr. Witchell, Mr. Melotte, and Mr. Stevens respectively.

Royal Observatory, Greenwich,
1900 *January* 12.

Ephemeris for Physical Observations

Greenwich Noon.	P.	L—O.	B.	A—L.	*B.*	Q.	E.
1900.	°	°	°	°	°	°	°
Aug. 24	338·43	193·51	+ 7·32	−28·06	−6·74	274·46	31·31
26	339·15	194·69	7·85	28·27	6·31	275·03	31·55
28	339·87	195·87	8·37	28·50	5·88	275·59	31·79
30	340·60	197·05	8·89	28·74	5·45	276·14	32·02
Sept. 1	341·33	198·23	9·40	28·96	5·02	276·68	32·25
3	342·06	199·40	+ 9·91	−29·19	−4·58	277·21	32·48
5	342·80	200·57	10·40	29·43	4·15	277·73	32·70
7	343·54	201·74	10·89	29·65	3·72	278·25	32·92
9	344·29	202·91	11·37	29·90	3·28	278·76	33·14
11	345·04	204·08	11·84	30·15	2·85	279·26	33·36
13	345·79	205·25	+12·30	−30·40	−2·42	279·75	33·57
15	346·54	206·41	12·76	30·64	1·99	280·24	33·78
17	347·29	207·57	13·21	30·89	1·56	280·72	33·99
19	348·04	208·73	13·65	31·14	1·14	281·19	34·20
21	348·79	209·89	14·08	31·39	0·71	281·65	34·40
23	349·55	211·05	+14·50	−31·65	−0·28	282·10	34·60
25	350·30	212·20	14·92	31·89	+0·14	282·55	34·80
27	351·05	213·35	15·33	32·15	0·56	282·98	34·99
29	351·80	214·50	15·73	32·41	0·98	283·40	35·18
Oct. 1	352·55	215·65	16·11	32·66	1·40	283·81	35·36
3	353·29	216·80	+16·48	−32·92	+1·82	284·21	35·54
5	354·03	217·94	16·85	33·17	2·24	284·59	35·71
7	354·77	219·08	17·21	33·43	2·65	284·97	35·87
9	355·51	220·22	17·55	33·68	3·07	285·34	36·03
11	356·24	221·35	17·89	33·92	3·48	285·70	36·18
13	356·97	222·48	+18·22	−34·17	+3·89	286·05	36·33
15	357·69	223·60	18·54	34·41	4·30	286·38	36·48
17	358·41	224·71	18·84	34·65	4·70	286·70	36·62
19	359·13	225·81	19·13	34·87	5·10	287·02	36·75
21	359·84	226·91	19·42	35·10	5·50	287·33	36·87
23	0·55	228·01	+19·69	−35·33	+5·90	287·62	36·99
25	1·24	229·10	19·96	35·55	6·30	287·90	37·10
27	1·92	230·19	20·22	35·76	6·69	288·17	37·20
29	2·60	231·27	20·46	35·97	7·08	288·43	37·29
31	3·27	232·34	+20·69	−36·17	+7·47	288·68	37·37

of Mars, 1900–01. By A. C. D. Crommelin.

Greenwich Noon.	⊙	Light Time.	Appar. Diam.	Defect of Illumination.	Central Meridian.	Passage of Zero Meridian.	
			q		ω_0		
1900.	°	m	″	″	°	h m	h m
Aug. 24	344·00	16·042	4·98	0·36	336·80	1 35·41	2 15·22
26		15·956	5·00	0·37	317·42	2 55·03	3 34·84
28		15·868	5·03	0·37	298·05	4 14·65	4 54·46
30		15·779	5·06	0·38	278·68	5 34·27	6 14·08
Sept. 1	348·17	15·689	5·09	0·39	259·31	6 53·89	7 33·70
3		15·597	5·12	0·40	239·94	8 13·51	8 53·32
5		15·503	5·15	0·41	220·57	9 33·12	10 12·92
7		15·408	5·18	0·42	201·21	10 52·71	11 32·50
9	352·28	15·311	5·21	0·43	181·85	12 12·29	12 52·08
11		15·213	5·25	0·44	162·49	13 31·87	14 11·66
13		15·113	5·28	0·45	143·13	14 51·44	15 31·23
15		15·012	5·32	0·46	123·77	16 11·01	16 50·79
17	356·33	14·909	5·35	0·46	104·42	17 30·56	18 10·33
19		14·806	5·39	0·47	85·07	18 50·10	19 29·86
21		14·699	5·43	0·48	65·72	20 9·61	20 49·36
23		14·590	5·47	0·49	46·37	21 29·11	22 8·86
25	0·34	14·480	5·51	0·50	27·03	22 48 61	23 28·36
27		14·368	5·56	0·51	7·69	...	0 8·10
29		14·255	5·60	0·51	348·35	0 47·85	1 27·61
Oct. 1		14·141	5·65	0·52	329·01	2 7·35	2 47·10
3	4·29	14·025	5·69	0·53	309·68	3 26·83	4 6·57
5		13·908	5·74	0·54	290·35	4 46·29	5 26·02
7		13·789	5·79	0·55	271·02	6 5·73	6 45·44
9		13·668	5·84	0·56	251·70	7 25·14	8 4·84
11	8 20	13·546	5·89	0·57	232·38	8 44·54	9 24·23
13		13·423	5·95	0·58	213·07	10 3·92	10 43·59
15		13·298	6·00	0·59	193·76	11 23·27	12 2·94
17		13·172	6·06	0·59	174·46	12 42·60	13 22·26
19	12·06	13·045	6·12	0 60	155·16	14 1·92	14 41·56
21		12·915	6·18	0·61	135·87	15 21·20	16 0·82
23		12·784	6·24	0·62	116·59	16 40·44	17 20·04
25		12·652	6·31	0·63	97·32	17 59·64	18 39·22
27	15·88	12·519	6·38	0·64	78·06	19 18·78	19 58·35
29		12·384	6·45	0·65	58·80	20 37·91	21 17·46
31		12·248	6·52	0·66	39·55	21 57·01	22 36·55

Greenwich Noon.	P.	L−O.	B.	A−L.	*B.*	Q.	E.
1900.	°	°	°	°	°	°	°
Nov. 2	3·93	233·40	+20·91	−36·36	+ 7·85	288·91	37·43
4	4·58	234·45	21·13	36·54	8·23	289·13	37·49
6	5·23	235·49	21·33	36·71	8·61	289·35	37·54
8	5·87	236·51	21·51	36·86	8·99	289·56	37·58
10	6·51	237·52	21·68	37·00	9·36	289·76	37·61
12	7·14	238·52	+21·85	−37·14	+ 9·73	289·94	37·62
14	7·71	239·51	22·01	37·26	10·10	290·11	37·63
16	8·30	240·49	22·15	37·37	10·47	290·27	37·61
18	8·88	241·44	22·29	37·46	10·83	290·42	37·58
20	9·45	242·38	22·42	37·53	11·19	290·56	37·53
22	10·01	243·31	+22·53	−37·59	+11·54	290·69	37·47
24	10·55	244·23	22·64	37·64	11·89	290·80	37·41
26	11·08	245·12	22·74	37·67	12·24	290·90	37·32
28	11·59	245·99	22·82	37·67	12·58	290·99	37·21
30	12·10	246·84	22·89	37·66	12·92	291·08	37·08
Dec. 2	12·58	247·68	+22·96	−37·63	+13·26	291·15	36·93
4	13·04	248·50	23·02	37·57	13·60	291·21	36·76
6	13·49	249·28	23·08	37·48	13·93	291·26	36·56
8	13·92	250·03	23·12	37·36	14·25	291·30	36·35
10	14·34	250·75	23·15	37·21	14·57	291·33	36·13
12	14·74	251·44	+23·18	−37·02	+14·89	291·35	35·88
14	15·12	252·10	23·20	36·81	15·21	291·36	35·58
16	15·48	252·73	23·22	36·56	15·52	291·35	35·25
18	15·82	253·33	23·23	36·29	15·83	291·33	34·89
20	16·14	253·89	23·23	35·98	16·13	291·30	34·50
22	16·44	254·42	+23·22	−35·63	+16·43	291·26	34·09
24	16·71	254·91	23·20	35·24	16·72	291·20	33·64
26	16·96	255·36	23·17	34·80	17·01	291·13	33·17
28	17·18	255·78	23·14	34·34	17·30	291·05	32·65
30	17·38	256·15	23·10	33·82	17·58	290·96	32·10
1901							
Jan. 1	17·56	256·47	+23·06	−33·26	+17·86	290·86	31·50
3	17·72	256·74	23·02	32·64	18·14	290·74	30·88
5	17·86	256·98	22·98	31·99	18·41	290·59	30·21
7	17·96	257·16	22·93	31·27	18·67	290·41	29·49
9	18·02	257·29	22·87	30·51	18·93	290·20	28·73
11	18·06	257·36	+22·80	−29·69	+19·19	289·95	27·92
13	18·09	257·38	+22·73	−28·80	+19·44	289·67	27·06

Greenwich Noon.	⊙	Light Time.	Appar. Diam.	Defect of Illumination.	Central Meridian.	Passage of Zero Meridian.	
				q	ω_0		
1900.	°	m	″	″	°	h m	h m
Nov. 2		12·111	6·59	0·67	20·31	23 16·08	
4	19·65	11·973	6·67	0·68	1·07	...	23 55·60
6		11·835	6·74	0·69	341·85	1 14·59	0 35·11
8		11·695	6·82	0·70	322·64	2 33·52	1 54·06
10		11·555	6·91	0·71	303·44	3 52·40	3 12·96
12	23·40	11·413	6·99	0·72	284·26	5 11·22	4 31·81
14		11·270	7·08	0·73	265·09	6 29·99	5 50·61
16		11·126	7·17	0·74	245·94	7 48·70	7 9·36
18		10·982	7·27	0·74	226·80	9 7·35	8 28·03
20	27·10	10·837	7·37	0·75	207·68	10 25·93	9 46·65
22		10·691	7·47	0·75	188·57	11 44·43	11 5·19
24		10·544	7·57	0·76	169·48	13 2·83	12 23·64
26		10·397	7·68	0·76	150·41	14 21·16	13 42·00
28	30·77	10·249	7·79	0·77	131·36	15 39·45	15 0·32
30		10·101	7·90	0·77	112·33	16 57·63	16 18·56
Dec. 2		9·954	8·02	0·78	93·31	18 15·70	17 36·68
4		9·806	8·14	0·78	74·31	19 33·65	18 54·69
6	34·42	9·658	8·26	0·79	55·35	20 51·50	20 12·58
8		9·510	8·39	0·79	36·43	22 9·14	21 30·33
10		9·362	8·52	0·80	17·53	23 26·70	22 47·93
12		9·214	8·66	0·80	358·65	0 5·45	...
14	38·03	9·067	8·80	0·81	339·80	1 22·89	0 44·18
16		8·921	8·95	0·81	321·00	2 40·18	2 1·56
18		8·776	9·10	0·81	302·23	3 57·24	3 18·73
20		8·631	9·25	0·80	283·49	5 14·18	4 35·73
22	41·62	8·487	9·41	0·80	264·78	6 31·00	5 52·61
24		8·343	9·57	0·79	246·10	7 47·68	7 9·36
26		8·200	9·73	0·79	227·47	9 4·17	8 25·97
28		8·059	9·90	0·78	208·89	10 20·44	9 42·33
30	45·19	7·920	10·08	0·78	190·34	11 36·55	10 58·52
1901 Jan. 1		7·784	10·25	0·78	171·82	12 52·50	12 14·55
3		7·650	10·43	0·77	153·35	14 8·29	13 30·42
5		7·517	10·62	0·75	134·93	15 23·88	14 46·13
7	48·74	7·388	10·81	0·73	116·57	16 39·15	16 1·54
9		7·260	10·99	0·71	98·26	17 54·22	17 16·71
11		7·134	11·19	0·68	80·00	19 9·09	18 31·68
13		7·012	11·38	0·65	61·78	20 23·76	19 46·45
							21 1·02

Greenwich Noon.	P.	L−O.	B.	A−L.	B.	Q.	E.
1901.	°	°	°	°	°	°	°
Jan. 15	18·09	257·37	+22·66	−27·89	+19·69	289·35	26·15
17	18·04	257·29	22·58	26·91	19·93	289·00	25·19
19	17·97	257·14	22·51	25·86	20·17	288·62	24·18
21	17·86	256 94	22·43	24·76	20·40	288·20	23·12
23	17·73	256·69	22·35	23·60	20·62	287·75	22·01
25	17·57	256·39	+22·27	−22·40	+20·84	287·25	20·84
27	17·38	256·01	22·18	21·11	21·06	286·68	19·63
29	17·16	255·57	22·09	19·75	21·27	286·02	18·37
31	16·90	255·09	22·00	18·35	21·48	285·24	17·06
Feb. 2	16·59	254·56	21·90	16·91	21·68	284·28	15·71
4	16·27	253·99	+21·80	−15·42	+21·88	282·99	14·32
6	15·93	253·36	21·69	13·87	22·07	282	12·89
8	15·56	252·68	21·58	12·26	22·26	280	11·42
10	15·15	251·97	21·47	10·63	22·44	277	9·92
12	14·73	251·23	21·36	8·97	22·62	274	8·42
14	14·29	250·47	+21·25	− 7·28	+22·79	270	6·92
16	13·83	249·67	21·13	5·55	22·95	264	5·45
18	13·35	248·85	21·01	3·80	23·11	252	4·08
20	12·87	248·00	20·90	2·01	23·26	231	3·01
22	12·38	247·16	20·79	− 0·23	23·41	197	2 63
24	11·89	246·33	+20·69	+ 1·54	+23·56	166	3·21
26	11·40	245·50	20·59	3·31	23·70	147	4·38
28	10·91	244·69	20.49	5·07	23·84	136	5·78
Mar. 2	10·43	243·89	20·40	6·81	23·97	130	7·27
4	9·97	243·10	20·30	8·55	24·09	126	8·78
6	9·52	242·35	+20·21	+10·25	+24·20	124	10·29
8	9·10	241·64	20·14	11·91	24·30	122	11·79
10	8·69	240·96	20·08	13·54	24·40	120	13·27
12	8·30	240·32	20·02	15·13	24·49	118·85	14·70
14	7·93	239·72	19·97	16·69	24·58	117·65	16·12
16	7·58	239·18	+19 93	+18·18	+24·66	116·68	17·48
18	7·28	238·69	19·91	19·63	24·74	115·88	18·79
20	7·01	238·25	19·90	21·03	24·82	115·18	20·05
22	6·77	237·85	19·90	22·39	24·89	114·53	21·29
24	6·56	237·52	19·91	23·69	24·95	113·97	22·45
26	6·38	237·25	+19·93	+24·92	+25·01	113·48	23·57
28	6·23	237·03	19·97	26·11	25·06	113·06	24·64
30	6·12	236·86	+20·02	+27·25	+25·10	112·70	25·65

Greenwich Noon.	⊙	Light Time.	Appar. Diam.	Defect of Illumination.	Central Meridian.	Passage of Zero Meridian.	
				q	ω_0		
1901.	°	m	″	″	°	h m	h m
Jan. 15	52·28	6·894	11·58	0·61	43·62	21 38·21	22 15·37
17		6·779	11·78	0·57	25·52	22 52·46	23 29·48
19		6·668	11·97	0·53	7·47	...	0 6·43
21		6·561	12·17	0·49	349·47	0 43·30	1 20·11
23	55·80	6·458	12·36	0·45	331·53	1 56·85	2 33·52
25		6·361	12·55	0·41	313·65	3 10·11	3 46·65
27		6·269	12·73	0·37	295·85	4 23·15	4 59·59
29		6·182	12·91	0·33	278·09	5 35·98	6 12·32
31	59·31	6·101	13·08	0·29	260·37	6 48·61	7 24·86
Feb. 2		6·024	13·25	0·25	242·70	8 1·05	8 37·19
4		5·954	13·41	0·21	225·07	9 13·25	9 49·27
6		5·892	13·55	0·18	207·51	10 25·26	11 1·21
8	62·81	5·836	13·68	0·14	189·99	11 37·12	12 12·99
10		5·784	13·80	0·11	172·49	12 48·82	13 24·61
12		5·740	13·91	0·08	155·02	14 0·36	14 36·07
14		5·704	14·00	0·05	137·57	15 11·78	15 47·46
16	66·30	5·676	14·07	0·03	120·16	16 23·12	16 58·76
18		5·656	14·12	0·01	102·78	17 34·39	18 10·01
20		5·642	14·15	0·00	85·40	18 45·62	19 21·22
22		5·634	14·17	0·00	68·03	19 56·82	20 32·42
24	69·80	5·634	14·17	0·00	50·64	21 8·03	21 43·67
26		5·643	44·15	0·01	33·25	22 19·33	22 55·01
28		5·660	14·10	0·02	15·84	23 30·71	...
Mar. 2		5·684	14·04	0·04	358·41	0 6·43	0 42·17
4	73·29	5·715	13·97	0·07	340·97	1 17·93	1 53·72
6		5·752	13·88	0·11	323·52	2 29·54	3 5·41
8		5·797	13·77	0·15	306·04	3 41·31	4 17·27
10		5·850	13·65	0·19	288·49	4 53·27	5 29·34
12	76·78	5·908	13·51	0·23	270·89	6 5·45	6 41·62
14		5·972	13·36	0·28	253·24	7 17·83	7 54·10
16		6·041	13·21	0·33	235·54	8 30·40	9 6·76
18		6·118	13·05	0·38	217·79	9 43·19	10 19·67
20	80·28	6·200	12·88	0·42	199·99	10 56·21	11 32·81
22		6·286	12·71	0·46	182·14	12 9·48	12 46·21
24		6·376	12·53	0·50	164·24	13 23·00	13 59·85
26		6·470	12·34	0·54	146·28	14 36·76	15 13·73
28	83·79	6·570	12·15	0·57	128·26	15 50·75	16 27·83
30		6·675	11·96	0·60	110·19	17 4·97	17 42·16

Greenwich Noon.	P.	L—O.	B.	A—L.	*B.*	Q.	E.
1901.	°	°	°	°	°	°	°
Apr. 1	6·05	236·74	+20·09	+28·3	+425·13	112·36	26·61
3	6·01	236·67	20·15	29·38	25·16	112·06	27·53
5	5·99	236·65	20·22	30·38	25·18	111·80	28·40
7	6·01	236·68	20·31	31·32	25·20	111·58	29·23
9	6·05	236·76	20·41	32·22	25·21	111·39	30·00
11	6·12	236·89	+20·52	+33·06	+25·21	111·23	30·74
13	6·23	237·07	20·64	33·86	25·21	111·10	31·43
15	6·38	237·30	20·77	34·61	25·20	111·00	32·08
17	6·55	237·57	20·91	35·32	25·18	110·92	32·69
19	6·74	237·89	21·05	35·98	25·16	110·86	33·26
21	6·96	238·24	+21·20	+36·61	+25·13	110·81	33·79
23	7·20	238·63	21·36	37·20	25·10	110·78	34·29
25	7·45	239·06	21·52	37·75	25·06	110·76	34·76
27	7·73	239·53	21·68	38·26	25·01	110·76	35·19
29	8·03	240·03	21·85	38·74	24·96	110·79	35·59
May 1	8·36	240·57	+22·02	+39·18	+24·90	110·83	35·95
3	8·70	241·14	22·20	39·60	24·83	110·88	36·29
5	9·06	241·73	22·38	39·99	24·75	110·93	36·61
7	9·44	242·37	22·56	40·34	24·67	110·98	36·90
9	9·83	243·03	22·74	40·66	24·58	111·03	37·16
11	10·24	243·73	+22·93	+40·95	+24·49	111·08	37·39
13	10·67	244·46	23·12	41·21	24·39	111·14	37·61
15	11·11	245·21	23·30	41·45	24·28	111·21	37·80
17	11·56	245·99	23·48	41·66	24·17	111·29	37·97
19	12·03	246·80	23·66	41·83	24·05	111·37	38·13
21	12·51	247·63	+23·83	+41·99	+23·92	111·46	38·28
23	13·00	248·48	24·00	42·13	23·78	111·56	38·41
25	13·49	249·35	24·16	42·25	23·64	111·65	38·50
27	14·00	250·24	24·33	42·34	23·49	111·74	38·56
29	14·53	251·16	24·49	42·41	23·34	111·84	38·62
31	15·06	252·11	+24·65	+42·44	+23·18	111·94	38·67
June 2	15·60	253·08	24·80	42·46	23·01	112·04	38·71
4	16·15	254·06	24·95	42·46	22·84	112·14	38·74
6	16·71	255·06	25·09	42·44	22·66	112·23	38·75
8	17·27	256·08	25·22	42·40	22·48	112·33	38·75
10	17·84	257·12	+25·35	+42·35	+22·29	112·42	38·73
12	18·41	258·18	25·47	42·27	22·09	112·51	38·70
14	18·98	259·25	+25·58	+42·19	+21·88	112·60	38·67

Greenwich Noon. 1901.	°	Light Time. m	Appar. Diam. ″	Defect of Illumination. q ″	Central Meridian. ω_0 °	Passage of Zero Meridian. h m	h m
Apr. 1		6·784	11·77	0·63	92·07	18 19·41	18 56·72
3		6·894	11·58	0·66	73·90	19 34·08	20 11·50
5	87·31	7·006	11·39	0·69	55·67	20 48·97	21 26·50
7		7·124	11·20	0·71	37·39	22 4·08	22 41·72
9		7·244	11·02	0·74	19·06	23 19·41	23 57·16
11		7·366	10·84	0·76	0·68	...	0 34·96
13	90·84	7·490	10·66	0·78	342·25	1 12·81	1 50·71
15		7·614	10·48	0·80	323·79	2 28·66	3 6·66
17		7·741	10·30	0·82	305·28	3 44·70	4 22·78
19		7·870	10·13	0·84	286·73	5 0·91	5 39·08
21	94·39	8·001	9·96	0·85	268·13	6 17·30	6 55·56
23		8·134	9·80	0·86	249·48	7 33·87	8 12·22
25		8·269	9·65	0·87	230·79	8 50·61	9 29·04
27		8·404	9·49	0·87	212·07	10 7·51	10 46·01
29	97·95	8·540	9·34	0·88	193·32	11 24·55	12 3·12
May 1		8·676	9·20	0·88	174·54	12 41·72	13 20·36
3		8·813	9·06	0·88	155·73	13 59·04	14 37·75
5		8·950	8·92	0·89	136·88	15 16·50	15 55·29
7	101·54	9·088	8·78	0·89	118·00	16 34·12	17 12·97
9		9·226	8·65	0·88	99·08	17 51·86	18 30·78
11		9·364	8·52	0·88	80·13	19 9·74	19 48·73
13		9·502	8·40	0·88	61·15	20 27·75	21 6·80
15	105·15	9·640	8·28	0·87	42·15	21 45·88	22 24·98
17		9·778	8·16	0·87	23·12	23 4·11	23 43·27
19		9·916	8·05	0·87	4·07	...	0 22·45
21		10·054	7·94	0·86	344·99	1 1·66	1 40·90
23	108·79	10·192	7·83	0·86	325·89	2 20·16	2 59·45
25		10·329	7·73	0·85	306·76	3 38·77	4 18·12
27		10·466	7·63	0·84	287·61	4 57·48	5 36·86
29		10·602	7·53	0·83	268·44	6 16·26	6 55·69
31	112·45	10·738	7·43	0·82	249·25	7 35·14	8 14·61
June 2		10·873	7·34	0·81	230·04	8 54·10	9 33·61
4		11·006	7·25	0·80	210·81	10 13·14	10 52·69
6		11·139	7·16	0·79	191·56	11 32·26	12 11·85
8	116·15	11·271	7·08	0·78	172·29	12 51·46	13 31·10
10		11·403	7·00	0·77	153·01	14 10·76	14 50·44
12		11·534	6·92	0·76	133·70	15 30·14	16 9·86
14		11·664	6·84	0·75	114·38	16 49·60	17 29·34

Greenwich Noon.	P.	L−O.	B.	Λ−L.	*B.*	Q.	E.
1901.	°	°	°	°	°	°	°
June 16	19·55	260·34	+25·68	+42·08	+21·67	112·68	38·62
18	20·13	261·44	25·77	41·96	21·45	112·75	38·56
20	20·70	262·55	25·86	41·83	21·23	112·82	38·49
22	21·28	263·68	25·93	41·68	21·00	112·89	38·41
24	21·86	264·82	26·00	41·52	20·77	112·95	38·32
26	22·44	265·98	+26·06	+41·34	+20·53	113·01	38·23
28	23·03	267·16	26·10	41·14	20·28	113·06	38·12
30	23·62	268·36	26·14	40·92	20·02	113·12	38·00
July 2	24·21	269·56	26·16	40·70	19·76	113·17	37·88
4	24·79	270·77	26·17	40·47	19·49	113·22	37·75
6	25·37	271·99	+26·18	+40·23	+19·22	113·25	37·62
8	25·94	273·22	26·17	39·98	18·94	113·27	37·48
10	26·51	274·46	26·15	39·71	18·66	113·29	37·33
12	27·08	275·71	26·12	39·44	18·37	113·30	37·17
14	27·65	276·97	26·08	39·15	18·07	113·31	37·01
16	28·21	278·24	+26·02	+38·86	+17·77	113·31	36·85
18	28·76	279·52	25·95	38·55	17·46	113·30	36·68
20	29·30	280·80	25·86	38·25	17·15	113·29	36·51
22	29·83	282·09	25·77	37·93	16·83	113·27	36·33
24	30·36	283·39	25·67	37·60	16·50	113·25	36·14
26	30·88	284·69	+25·55	+37·28	+16·17	113·21	35·95
28	31·38	286·00	25·42	36·94	15·84	113·16	35·75
30	31·87	287·31	25·28	36·60	15·50	113·11	35·55
Aug. 1	32·35	288·63	25·12	36·25	15·15	113·04	35·34
3	32·83	289·96	24·95	35·89	14·80	112·96	35·13
5	33·30	291·29	+24·76	+35·53	+14·44	112·88	34·92
7	33·75	292·62	24·56	35·18	14·08	112·79	34·71
9	34·19	293·95	24·35	34·82	13·72	112·69	34·49
11	34·61	295·29	24·12	34·46	13·35	112·57	34·27
13	35·01	296·63	23·88	34·09	12·97	112·45	34·04
15	35·39	297·97	+23·62	+33·73	+12·59	112·32	33·81
17	35·76	299·31	23·35	33·36	12·20	112·18	33·57
19	36·11	300·66	23·08	32·98	11·81	112·03	33·33
21	36·44	302·01	22·79	32·61	11·41	111·87	33·09
23	36·76	303·36	22·49	32·23	11·01	111·70	32·85
25	37·06	304·71	+22·17	+31·86	+10·60	111·52	32·60
27	37·35	306·06	21·84	31·48	10·19	111·33	32·35
29	37·61	307·40	+21·50	+31·12	+9·78	111·13	32·10

Greenwich Noon.	⊙	Light Time.	Appar. Diam.	Defect of Illumination. q	Central Meridian. ω_0	Passage of Zero Meridian.	
1901.	°	m	″	″	°	h m	h m
June 16	119·88	11·793	6·77	0·74	95·05	18 9·09	
18		11·921	6·70	0·73	75·71	19 28·64	18 48·86
20		12·048	6·63	0·72	56·35	20 48·24	20 8·43
22		12·174	6·56	0·71	36·97	22 7·91	21 28·07
24	123·65	12·300	6·49	0·70	17·57	23 27·64	22 47·77
26		12·424	6·42	0·69	358·16	0 7·53	...
28		12·547	6·36	0·68	338·73	1 27·37	0 47·44
30		12·669	6·30	0·67	319·29	2 47·28	2 7·32
July 2	127·46	12·790	6·24	0·66	299·84	4 7·25	3 27·26
4		12·909	6·18	0·65	280·39	5 27·29	4 47·26
6		13·027	6·12	0·64	260·92	6 47·35	6 7·32
8		13·144	6·07	0·63	241·45	8 7·44	7 27·39
10	131·31	13·260	6·02	0·62	221·96	9 27·55	8 47·49
12		13·375	5·97	0·61	202·47	10 47·67	10 7·60
14		13·489	5·92	0·60	182·97	12 7·83	11 27·75
16		13·601	5·87	0·59	163·46	13 28·02	12 47·92
18	135·21	13·711	5·82	0.58	143·94	14 48·26	14 8·13
20		13·821	5·78	0·57	124·41	16 8·57	15 28·41
22		13·930	5·73	0·56	104·88	17 28·92	16 48·74
24		14·037	5·69	0·54	85·34	18 49·31	18 9·11
26	139·16	14·143	5·65	0·53	65·79	20 9·71	19 29·51
28		14·248	5·61	0·52	46·24	21 30·11	20 49·91
30		14·351	5·57	0·51	26·68	22 50·52	22 10·31
Aug. 1		14·453	5·53	0·50	7·12	...	23 30·73
3	143·15	14·553	5·49	0·49	347·56	0 51·15	0 10·94
5		14·652	5·45	0·49	328·00	2 11·59	1 31·37
7		14·750	5·41	0·48	308·43	3 32·06	2 51·82
9		14·847	5·38	0·47	288·86	4 52·55	4 12·30
11	147·20	14·943	5·34	0·46	269·28	6 13·06	5 32·80
13		15·037	5·31	0·45	249·69	7 33·58	6 53·32
15		15·129	5·27	0·45	230·11	8 54·10	8 13·84
17		15·220	5·24	0·44	210·53	10 14·61	9 34·35
19	151·30	15·310	5·21	0·43	190·94	11 35·12	10 54·87
21		15·399	5·18	0·42	171·36	12 55·64	12 15·38
23		15·487	5·15	0·42	151·78	14 16·15	13 35·89
25		15·573	5·12	0·41	132·20	15 36·67	14 56·41
27	155·45	15·658	5·09	0·40	112·62	16 57·19	16 16·93
29		15·742	5·07	0·39	93·03	18 17·72	17 37·46
							18 57·98

Greenwich Noon.	P.	L−C.	B.	Λ−L.	*B.*	Q.	E.
1901.	°	°	°	°	°	°	°
Aug. 31	37·86	308·75	+21·14	+30·75	+ 9·36	110·91	31·85
Sept. 2	38·08	310·09	20·77	30·39	8·94	110·68	31·59
4	38·29	311·44	20·38	30·02	8·51	110·44	31·33
6	38·48	312·79	19·99	29·65	8·08	110·19	31·07
8	38·64	314·13	19·58	29·29	7·65	109·93	30·81
10	38·78	315·47	+19·16	+28·93	+ 7·22	109·66	30·55
12	38·88	316·80	18·72	28·59	6·78	109·37	30·28
14	38·96	318·11	18·26	28·26	6·33	109·07	30·01
16	39·01	319·41	+17·79	+27·95	+ 5·88	108·76	29·74

This ephemeris has been computed with the same constants as those for the last two oppositions. The position of the north pole of *Mars* is assumed to be R.A. 317°·17, N.P.D. 37°·42 at the beginning of the ephemeris, and R.A. 317°·18, N.P.D. 37°·41 at the end of it. These values are those deduced by Struve from observations of the satellites. I have continued to use them in the absence of any suggestions to the contrary.

P. denotes the position angle of the axis of *Mars*; L−O+180° the Right Ascension of the Earth referred to the plane of *Mars*' equator, and reckoned from O, the point of the vernal equinox of *Mars*' northern hemisphere; Λ−O+180°, the corresponding quantity for the Sun; B, *B*, the declinations of the Earth and Sun reckoned from the plane of *Mars*' equator; Q, the position angle of the greatest defect of illumination; *q*, the amount of the greatest defect of illumination; E, the areocentric angle between the Earth and Sun.

Thus Λ−L is the angle between the meridians of *Mars* which are central to the Earth and to the Sun; the point where an image of the Sun should be looked for is distant $\frac{1}{2}(\Lambda-L)$ in longitude from the centre of the disc, on the side towards the fully illuminated limb, and is in areocentric latitude $\frac{1}{2}(B+B)$.

The column ⊙, given now for the first time, is the areocentric longitude of the Sun referred to the plane of *Mars*' orbit, and reckoned from O, the point of the vernal equinox of *Mars*' northern hemisphere. The object of this column is to ascertain readily the season on *Mars*. ⊙=0° corresponds to March 20 on the Earth, ⊙=30° to April 20, and so on.

The diameter of the planet at distance unity is assumed (as before) to be 9″·60, that used in the *Nautical Almanac* being 9″·36.

The zero meridian and period of rotation have been carried on unchanged from Mr. Marth's last ephemeris, the assumed sidereal rotation in 24 hours being 350°·89202. I should be glad to have observations of the transits of any well-determined

Greenwich Noon. 1901.	⊙	Light Time.	Appar. Diam.	Defect of Illumination. q	Central Meridian. ω_0	Passage of Zero Meridian.	
	°	m	″	″	°	h m	h m
Aug. 31		15·824	5·04	0·38	73·45	19 38·24	20 18·51
Sept. 2		15·905	5·02	0·37	53·86	20 58·77	21 39·02
4	159·66	15·985	4·99	0·37	34·28	22 19·27	22 59·52
6		16·063	4·97	0·36	14·71	23 39·77	...
8		16·140	4·94	0·35	355·13	0 20·02	1 0·26
10		16·216	4·92	0·34	335·56	1 40·51	2 20·75
12	163·93	16·292	4·90	0·33	315·98	3 1·00	3 41·24
14		16·367	4·88	0·33	296·41	4 21·48	5 1·72
16		16·440	4·86	0·32	276·84	5 41·96	

points across the centre of the disc communicated to me, as a check on the accuracy of the values assumed in the ephemeris.

The last two columns contain the Greenwich Mean Time of transit of the zero meridian across the centre of the disc, the times being given for the intermediate days as well as for those given in the date column. They are tabulated to $0^m{\cdot}01$, but the last figure given is only approximate.

The quantities are to be interpolated directly from the ephemeris for the time for which they are required, the equation of light having been already applied.

The vernal equinox of *Mars'* northern hemisphere occurs 1900 September $24^d{\cdot}3$ and the summer solstice 1901 April $11^d{\cdot}1$.

Benvenue, 55 *Ulundi Road, Blackheath, S.E.:*
1900 *January* 10.

Erratum in Prof. E. W. Brown's paper, vol. lx. p. 124, lines 18, 19,

for "parts of the ratio of the mean motions,"
read "mean motions of the perigee and node."

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